



**Calhoun: The NPS Institutional Archive**  
**DSpace Repository**

---

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

---

1966

# Theoretical analysis of the heated jet pump and design of a test facility/

Ridder, William Clayton

Monterey, California. U.S. Naval Postgraduate School

---

<http://hdl.handle.net/10945/28602>

---

*Downloaded from NPS Archive: Calhoun*



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

**Dudley Knox Library / Naval Postgraduate School**  
**411 Dyer Road / 1 University Circle**  
**Monterey, California USA 93943**

<http://www.nps.edu/library>

**THEORETICAL ANALYSIS OF THE HEATED  
JET PUMP AND DESIGN OF A TEST FACILITY**

**WENDELL CLAYTON RIDDER  
and  
CARL RICHARDS SUMMERS**

1. 1. 1.  
NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CALIF. 93940







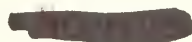
THEORETICAL ANALYSIS OF THE  
HEATED JET PUMP AND  
DESIGN OF A TEST FACILITY

by

Wendell Clayton Ridder  
Lieutenant, United States Navy  
B.S., University of Kansas, 1960

and

Carl Richards Summers  
Lieutenant, United States Navy  
B.S., University of Colorado, 1958



Submitted in partial fulfillment  
for the degree of

MASTER OF SCIENCE

IN

AERONAUTICAL ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL  
May 1966

TW515  
H 15  
- 211

## ABSTRACT

The jet pump is a device that compresses low-pressure fluid by means of a high energy jet. This device is simple, reliable and light weight, and is attractive for various uses including eventual application to boundary layer control for aircraft.

In this study the theoretical flow of a perfect gas through constant pressure and constant area jet pumps is predicted by analyzing the equations of continuity, energy and momentum. Of particular interest is the effect of heating the high energy jet. The parameters describing optimum heated jet pumps are determined. The ideal pumps presented herein represent upper performance limits for actual devices.

Also included is a complete description and design of a facility for testing heated jet pumps. Specific test configurations are analyzed, and performance curves are illustrated. These curves facilitate comparison between theory and experiment.

# TABLE OF CONTENTS

Section	Page
1. Introduction	7
2. Theoretical Analysis	9
Definition of Performance Coefficients	9
Derived Coefficients	10
Efficiency	11
Constant Pressure Development	12
Constant Area Development	14
Discussion of Results	18
Fixed Area - Variable Discharge Pressure	20
Theoretical Conclusions	22
3. Test Facility Design	23
Primary Air Supply System	23
Secondary Air System	25
Mixing Tube	25
Instrumentation	27
Mass Flow Measurement	28
Cost Estimate	29
4. Recommendations for Further Study	31
Bibliography	66
Appendix A - Detailed Development	67
Appendix B - Itemized Cost Estimate	89
Appendix C - Computer Results	101

## LIST OF ILLUSTRATIONS

Figure	Page
1. Typical Constant Pressure Jet Pump Enthalpy/Entropy Diagram	32
2. Typical Constant Area Jet Pump Enthalpy/Entropy Diagram	33
3. Predicted Performance Curves for Two Families of Constant Area Pumps	34
4-6. Maps of Cpm Compared to Cqm for Optimum Constant Pressure Jet Pumps	35
7-9. Maps of Cpm Compared to Cqm for Optimum Constant Area Jet Pumps	38
10-12. Performance Maps for Optimum Constant Pressure Jet Pumps	41
13-15. Performance Maps for Optimum Constant Area Jet Pumps	44
16-24. Curves for Fixed Area Ratio - Variable Discharge Pressure Jet Pump Operation	47
25. General Design Concept for a Constant Area Heated Jet Pump	57
26-28. Nozzle and Mixing Tube Entrance Designs	58
29. Secondary Air Plenum Chamber Design	61
30. Mixing Tube Stanchions and Supports	62
31. Mixing Tube Handling Cart	63
32. Mixing Tube Instrumentation	64
33. Mass Flow Measurement Design	65

# TABLE OF SYMBOLS

## Symbols

$A$	cross-sectional area
$a^*$	speed of sound at Mach Number equal unity
$c_p$	specific heat at constant pressure
$c_v$	specific heat at constant volume
$C_M$	momentum coefficient
$C_P$	power coefficient
$C_{PM}$	power-momentum coefficient
$C_Q$	heat coefficient
$C_{QM}$	heat-momentum coefficient
$D$	diameter of duct
$g$	acceleration given a unit mass by unit force
$h$	static enthalpy
$H$	total enthalpy
$L$	length of duct
$\dot{m}$	mass flow rate
$M^*$	velocity ratio, $V/a^*$
$p$	static pressure
$P_t$	total pressure
$q$	heat
$R$	gas constant
$s$	entropy
$T$	absolute temperature
$V$	velocity

# Table of Symbols (Cont'd)

## Symbol

$\alpha$	mass flow ratio, $\dot{m}_2/\dot{m}_3$
$\gamma$	area ratio, $A_2/A_3$
$\gamma$	ratio of specific heats, $c_p/c_v$
$\Delta$	difference or change of
$\eta$	efficiency
$\rho$	density
$\phi(M^*)$	defined velocity function
$\psi(M^*)$	defined velocity function
$( )_t$	signifies stagnation state
$( )_2$	signifies station location, numbers 1 through 6
$( )^*$	signifies the state at which the Mach Number is unity; does not apply to $M^*$



## 1. Introduction

A jet pump is a device designed to draw a given mass flow of fluid from a low-pressure source and deliver it to a region of higher pressure by means of a high energy jet of air. Basically, this analysis was undertaken to investigate the theoretical operation of jet pumps, and to recommend a test facility design to provide for the comparison of theoretical and experimental data. The ultimate objective of this investigation is the eventual application of jet pumps to boundary layer control on aircraft.

The jet pump is attractive for an application of this nature due to its reliability, simplicity, and light weight. Also, this device can be readily incorporated into a jet aircraft's power plant cycle. For certain conditions, such as full-throttle operation, it is conceivable that the mechanical power available to the jet pump may be limited. Hence, it is of interest to investigate if the mechanical power required for a given application can be reduced by heating the primary jet air.

The theoretical analysis of ideal jet pump performance contained herein considers the effect of this heating. Constant area and constant pressure mixing processes are investigated. In all cases it is possible to predict "optimum conditions" for specified jet pump configurations. Since all losses except mixing losses are neglected, the results of this study represent a performance limit that actual jet pumps can approach, but never exceed. The final results shown in Appendix C were determined by means of the Control Data Corporation 1604 digital computer. This theoretical analysis is an extension of the work initiated by

Belter in Reference 1.

For any study to be complete it is necessary to substantiate the theoretical results by experimental data. In view of this fact, a design for a heated jet pump test facility is presented. Several test facility configurations along with their material cost estimates are included.

The writers wish to express their appreciation for the assistance and guidance provided by Professor Theodore J. Gawain, of the U. S. Naval Postgraduate School, Monterey, California.

## 2. Theoretical analysis

### Definition of performance coefficients

A jet pump must be able to produce a required discharge momentum. For this output, a certain amount of mechanical power and heat energy must be supplied to the jet air. Reference 1 expresses the power and heat inputs and discharge momentum as dimensionless coefficients. The arbitrary reference dimensions chosen were for ambient air at sonic velocity and the hypothetical area,  $A_6$ , necessary to pass the discharge from the jet pump at sonic velocity.

#### a. Momentum coefficient

The momentum coefficient is defined as the dimensionless discharge momentum.

$$C_M = \frac{\rho_5 A_5 V_5}{\rho_o^* A_6 a_o^*} \quad (1)$$

#### b. Power coefficient

The power coefficient is defined as the dimensionless power supplied to the primary air.

$$C_P = \frac{\rho_3 A_3 V_3}{\rho_o^* A_6 a_o^*} \frac{c_p \Delta T_c}{\frac{a_o^{*2}}{2gJ}} \quad (2)$$

where  $\Delta T_c$  is the temperature change through the compressor.

#### c. Heat coefficient

The heat coefficient is defined as the dimensionless heat supplied to the primary air.

$$C_Q = \frac{\rho_3 A_3 V_3}{\rho_o^* A_6 a_o^*} \frac{c_p \Delta T_B}{\frac{a_o^{*2}}{2gJ}} \quad (3)$$

where  $\Delta T_B$  is the temperature change through the burner.

#### Derived coefficients

It is convenient to compare the mechanical power and heat energy to some significant parameter of the jet pump in order to determine the effectiveness of any design. The reference parameter chosen was the discharge momentum of the jet pump. By dividing the power coefficient and the heat coefficient by the momentum coefficient, it is possible to obtain derived power-momentum and heat-momentum coefficients.

$$C_{PM} = \frac{C_P}{C_M} = \frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5} \frac{a_o^*}{V_5} \frac{c_p \Delta T_c}{\frac{a_o^{*2}}{2gJ}} \quad (4)$$

$$C_{QM} = \frac{C_Q}{C_M} = \frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5} \frac{a_o^*}{V_5} \frac{c_p \Delta T_B}{\frac{a_o^{*2}}{2gJ}} \quad (5)$$

It is shown in Appendix A that  $C_{PM}$ ,  $C_{QM}$ , and  $C_M$  can be reduced to the following:

$$C_M = \frac{P_{t5}}{P_{t0}} \left\{ \frac{\gamma+1}{\gamma-1} \left[ 1 - \frac{P_{t0}}{P_{t5}} \right]^{\frac{\gamma-1}{\gamma}} \right\}^{1/2} \quad (6)$$

$$\frac{C_{PM}}{\left[ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} = \frac{C_{QM}}{\left[ \frac{T_{t1}}{T_{t0}} - \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right]} = \frac{\left( \frac{\gamma+1}{\gamma-1} \right) \frac{1}{M_5^*}}{\sqrt{(\gamma-1) \left( \gamma + \frac{T_{t1}}{T_{t0}} \right)^\gamma}} \quad (7)$$

where  $\gamma$  is the mass flow ratio,  $\dot{m}_2/\dot{m}_3$ .

Equation (6) shows that  $C_M$  depends only on the discharge total pressure ratio,  $P_{t5}/P_{t0}$ . In particular,  $C_M$  is independent of the amount of heat supplied. By specifying the desired output pressure ratio,  $P_{t5}/P_{t0}$ , the required  $C_M$  is determined.

Consider the case where  $P_{t1}/P_{t0}$  ,  $T_{t1}/T_{t0}$  and  $P_{t5}/P_{t0}$  are fixed. Equation (7) shows that under these conditions  $C_{PM}$  and  $C_{QM}$  will be a minimum when the mass flow ratio,  $\alpha$  , is a maximum. It is noted that Equation (7) is invariant for both the constant pressure and constant area cases, although the maximum value of  $\alpha$  may differ in the two cases.

In a sense,  $C_{PM}$  is a measure of the compressor power required and  $C_{QM}$  is a measure of the heat energy required. Similarly,  $C_M$  is a measure of the demanded discharge momentum. Any improvement in design and performance should be reflected by a decrease in power and/or fuel requirements. Therefore, as may be seen from Equation (7), a necessary and sufficient criterion for optimum performance is that the mass flow ratio,  $\alpha$  , be a maximum. Henceforth, an optimum pump is defined as one that has the maximum possible  $\alpha$  for specified values of  $P_{t1}/P_{t0}$  ,  $T_{t1}/T_{t0}$  , and  $P_{t5}/P_{t0}$  . It will be shown later that under these conditions an optimum pump corresponds to a specific area ratio,  $A_2/A_3$  . Any other value of area ratio will produce less than the maximum possible mass flow ratio.

### Efficiency

Another performance parameter of some interest is the mechanical efficiency which is, of course, based on energy considerations, namely,

$$\eta_M \equiv \frac{\text{Final available energy (Isentropic)}}{\text{Initial available energy (Isentropic)}}$$

$$\eta_M = \frac{\dot{m}_5 (H_5 - h_5)}{\dot{m}_3 \Delta H_c}$$

It is shown in Appendix A that the above definition is reducible to the result:

$$\eta_M = \frac{\left\{ 1 - \left( \frac{P_{t0}}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right\}}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} \left( \chi + \frac{T_{t1}}{T_{t0}} \right) \quad (8)$$

From Equation (8),  $\eta_M$ , like the other performance parameters, will be optimum when  $\chi$  is a maximum for specified conditions of pressure and temperature ratio.

#### Constant pressure development

This analysis is an optimization of the performance of the constant pressure heated jet pumps which were described in Reference 1. A typical constant pressure mixing jet pump is shown in Figure 1 along with the associated h-s diagram. The basic development of the constant pressure case is from Reference 2. The many details of the development are contained in Appendix A.

The usual isentropic relationships were used to solve for conditions in the primary (jet) system and in the secondary system up to the mixing tube entrance. It is also assumed that the specific heats remain constant.

The three basic equations that have to be satisfied for constant pressure mixing (  $p_2 = p_4$  ) are continuity, energy, and momentum.

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \quad (9)$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \quad (10)$$

Momentum:

$$(\dot{m}_3 + \dot{m}_2) V_4 - (\dot{m}_3 V_3 + \dot{m}_2 V_2) = (p_3 - p_2) A_3 \quad (11)$$

It is noted that the resultant pressure forces in Equation (11) are zero when the velocity in the primary nozzle is subsonic.

The equations of continuity, energy, and momentum can be expressed in non-dimensional terms. This development is shown in Appendix A.

Continuity:

$$\frac{A_4}{A_3} = \sqrt{(1 + \chi) \left( \frac{T_{t1}}{T_{t0}} + \chi \right)} \left\{ \frac{\frac{P_{t1}}{P_{t0}}}{\left( \frac{P_{t5}}{P_{t0}} \right) \sqrt{\frac{T_{t1}}{T_{t0}}}} \right\} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*} \quad (12)$$

Energy:

$$\frac{T_{t1}}{T_{t0}} + \chi = (1 + \chi) \frac{T_{t4}}{T_{t0}} \quad (13)$$

Momentum:

$$\sqrt{\frac{T_{t1}}{T_{t0}}} M_3^* + \chi M_2^* + \sqrt{\frac{T_{t1}}{T_{t0}}} \xi = (1 + \chi) \sqrt{\frac{T_{t4}}{T_{t0}}} M_4^* \quad (14)$$

where,

$$\xi = \frac{1}{\gamma} \left\{ 1 - \left( \frac{P_{t0}}{P_{t1}} \right) \left[ \frac{1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}}{\frac{2}{\gamma+1}} \right]^{\frac{\gamma}{\gamma-1}} \right\}$$

By fixing  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ ,  $P_{t5}/P_{t0}$ , and  $M_2^*$ , it is possible to obtain an explicit solution from Equations (12), (13), and (14) since there remains only three unknowns  $\chi$ ,  $T_{t4}/T_{t0}$ , and  $A_4/A_3$ .

Equations (13) and (14) are solved simultaneously for  $\chi$  and  $T_{t4}/T_{t0}$ . By eliminating  $T_{t4}/T_{t0}$  between the two equations, the following quadratic equation in  $\chi$  is obtained.

$$(M_4^{*2} - M_2^{*2}) \chi^2 + \left\{ \left( \frac{T_{t1}}{T_{t0}} + 1 \right) M_4^{*2} - 2 \sqrt{\frac{T_{t1}}{T_{t0}}} M_2^{*2} (M_3^* + \xi) \right\} \chi + \frac{T_{t1}}{T_{t0}} \left\{ M_4^{*2} - (M_3^* + \xi)^2 \right\} = 0 \quad (15)$$

Equation (15) can be solved using the quadratic formula

$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



where

$$\begin{aligned} a &= M_4^{*2} - M_2^{*2} \\ b &= \left( \frac{T_{t1}}{T_{t0}} + 1 \right) M_4^{*2} - 2 \sqrt{\frac{T_{t1}}{T_{t0}}} M_2^* (M_3^* + \xi) \\ c &= \frac{T_{t1}}{T_{t0}} \left\{ M_4^{*2} - (M_3^* + \xi)^2 \right\} \end{aligned}$$

Since the mass flow ratio,  $\chi$ , must be positive and real, one limiting condition is that  $b^2 - 4ac \geq 0$ . For all cases investigated this condition is satisfied. Other limiting conditions are  $M_2^* \leq 1.0$  and  $M_3^* \leq 1.0$  since the primary and secondary nozzles are only converging, not converging-diverging. Both roots of Equation (15) were determined. One of these was negative and, therefore, was discarded as meaningless.

From the solution of Equation (15) it was possible to determine  $T_{t4}/T_{t0}$  from the energy Equation (13) and  $A_4/A_3$  from the continuity Equation (12).  $A_2/A_3$  can be solved directly from the mass flow ratio.

The computer was used to obtain solutions to Equations (12), (13), and (14). The parameters  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ ,  $P_{t5}/P_{t0}$ , and  $M_2^*$  were varied systematically to obtain families of constant pressure pumps. The optimum pump from each family was determined. Appendix C contains solutions for each optimum pump for the conditions specified.

#### Constant Area Development

This analysis is an investigation and optimization of the ideal flow of a perfect gas through a constant area jet pump. Figure 2 shows a sketch and a general h-s diagram of a constant area pump. This figure indicates the notation to be used to denote station locations throughout this development.

Much as before, this analysis involves the solution of the continuity, energy, and momentum equations. These equations may be written in absolute form as follows:

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \quad (16)$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \quad (17)$$

Momentum:

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \quad (18)$$

It is shown in Appendix A that the above equations can be non-dimensionalized to yield the following relationships:

Continuity:

$$y \phi(M_2^*) + \frac{P_{t1}}{P_{t0}} \sqrt{\frac{T_{t0}}{T_{t1}}} \phi(M_3^*) = (1+y) \frac{P_{t4}}{P_{t0}} \sqrt{\frac{T_{t0}}{T_{t4}}} \phi(M_4^*) \quad (19)$$

Energy:

$$y \phi(M_2^*) + \frac{P_{t1}}{P_{t0}} \sqrt{\frac{T_{t1}}{T_{t0}}} \phi(M_3^*) = (1+y) \frac{P_{t4}}{P_{t0}} \sqrt{\frac{T_{t0}}{T_{t4}}} \phi(M_4^*) \quad (20)$$

Momentum:

$$y \psi(M_2^*) + \frac{P_{t1}}{P_{t0}} \psi(M_3^*) = (1+y) \frac{P_{t4}}{P_{t0}} \psi(M_4^*) \quad (21)$$

where:

$$\phi(M^*) = \left(1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{\frac{1}{\gamma-1}} M^*$$

$$\psi(M^*) = \left(1 - \frac{\gamma-1}{\gamma+1} M^{*2}\right)^{\frac{1}{\gamma-1}} (1 + M^{*2})$$

and  $y$  is the area ratio,  $A_2/A_3$ .

The development of Equations (19), (20) and (21) is from Reference

2. A similar development is included in Reference 1. However, an error

was found in Equation (37) of Reference 1 rendering incorrect solutions for the constant area case presented therein.

It is assumed that the parameters  $P_{t1}/P_{t0}$  ,  $T_{t1}/T_{t0}$  , and  $P_{t5}/P_{t0}$  are given. Values of the secondary velocity ratio,  $M_2^*$  , are systematically chosen. Once  $M_2^*$  is fixed the primary velocity ratio,  $M_3^*$  , can be determined as shown in Appendix A. The remainder of the problem resolves into a solution of Equations (19), (20) and (21) for the unknowns  $M_4^*$  ,  $y$  and  $T_{t4}/T_{t0}$  ; and satisfying the boundary condition that the value of  $P_{t4}/P_{t0}$  so obtained must equal the value originally specified for  $P_{t5}/P_{t0}$  .

Since the resulting relationships are transcendental, it is necessary to solve the equations by iterative procedures. It is thereby possible to achieve solutions for numerous families of constant area heated jet pumps.

The routine employed to solve the equations is to choose incremental values for  $M_2^*$  . Choosing  $M_2^*$  fixes all the known parameters not previously specified. A trial value of the area ratio,  $y$ , is assumed. The mass flow ratio is then solved by the relationship:

$$\chi = \dot{m}_2/\dot{m}_3 = y \frac{P_{t0}/\sqrt{T_{t1}}}{P_{t1}/\sqrt{T_{t0}}} \frac{\phi(M_2^*)}{\phi(M_3^*)} \quad (22)$$

Now it is possible to solve for the discharge total temperature ratio,  $T_{t4}/T_{t0}$  , by using the equation:

$$\frac{T_{t4}}{T_{t0}} = \frac{\chi + T_{t1}/T_{t0}}{\chi + 1} \quad (23)$$

This procedure determines all the parameters except the exit velocity ratio,  $M_4^*$  . The final equation solved is a quadratic in terms of  $M_4^*$  .

$$\frac{M_4^*}{1 + M_4^{*2}} = \frac{\sqrt{\frac{T_{t0}}{T_{t4}}} \frac{y \phi(M_2^*) + (P_{t4}/P_{t0}) \sqrt{\frac{T_{t4}}{T_{t0}}} \phi(M_3^*)}{y \psi(M_2^*) + (P_{t4}/P_{t0}) \psi(M_3^*)} \quad (24)$$

The above equation gives a supersonic and a subsonic solution for  $M_4^*$ . For this particular analysis (i.e.,  $M_2^*$  and  $M_3^* \leq 1$  and constant area mixing), the supersonic solution for  $M_4^*$  is meaningless. This means to say that  $M_4^*$  cannot be supersonic for a constant area pump when the inlet velocity ratios,  $M_2^*$  and  $M_3^*$ , are limited to one or less. However, the supersonic solution of  $M_4^*$  may be of interest if  $M_3^*$  and/or  $M_2^*$  are greater than one.

The preceding procedure gives a solution to the equations; however, it is also necessary to satisfy the boundary conditions. The resulting exit total pressure ratio,  $P_{t4}/P_{t0}$ , is checked against the desired discharge total pressure ratio,  $P_{t5}/P_{t0}$ . If these quantities are not equal, the area ratio is adjusted in order to achieve the necessary conditions at the end of the mixing section. This process is repeated to determine families of constant area jet pumps.

Figure 3 shows the performance curves for two families of constant area pumps. As defined previously, the optimum pumps are those which achieve a maximum mass flow ratio for given values of  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ , and  $P_{t5}/P_{t0}$ . It is observed that optimum conditions are reached in two different ways for the constant area case.

Again referring to Figure 3, curve A passes through a true mathematical maximum in the sense that  $\frac{dM_z^*}{dx} = 0$ . This curve shows that as  $M_2^*$  increases the mass flow ratio increases up to a maximum and then decreases. This situation will be referred to as case A hereafter.

Curve B shows the other limiting case of the constant area system.

This curve increases up to a point of maximum mass flow ratio and simply ceases. This phenomena is not clearly understood. Constant area pumps that reach optimum conditions in this manner will be known as case B.

The computer solutions in Appendix C contain a complete set of optimum constant area heated jet pumps for the conditions specified. The non-starred optimum pumps correspond to case A, and the starred pumps are limited in a manner illustrated by case B. These results will be discussed in a following section.

#### Discussion of Theoretical Results

One significant indication of the performance of a jet pump is the amount of energy that must be supplied to produce the discharge momentum demanded. The coefficients  $C_{PM}$  and  $C_{QM}$  are simply numbers indicating the energy supplied; and the coefficient  $C_M$  is a measure of the discharge momentum required. These coefficients show the effect of changing the parameters  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ , and  $P_{t5}/P_{t0}$ .

Figures 4 through 9 are graphs of  $C_{PM}$  compared to  $C_{QM}$  for families of optimum constant pressure and constant area jet pumps. These graphs are basically maps formed by curves of constant supply pressure and temperature ratios. One of the most significant results illustrated by these maps is the trade-off between the heat energy and compressor power necessary to produce the desired  $C_M$ .

For example, consider Figure 4. If the supply pressure ratio is fixed,  $C_{PM}$  decreases as the supply temperature ratio increases. This indicates that the compressor power required is reduced by heating the primary air. Conversely, for a constant temperature ratio,  $C_{QM}$  is



reduced as  $P_{t1}/P_{t0}$  is increased. For the highest temperature ratio and the lowest pressure ratio, the compressor power is minimal. It is also observed that the greatest incremental decrease in  $C_{PM}$  occurs between temperature ratios of 1 and 2 along lines of constant pressure.

Now consider the sequence of Figures 4 through 6. It can be seen that as the momentum coefficient demanded is increased, the maps tend to move up and to the right. This, of course, corresponds to higher supply energy requirements. Also, it is noted that the effect of heating is greater as the demand on the pump is increased.

Figures 10 through 15 are graphs of maximum mass flow ratio versus  $M_2^*$ . These maps illustrate the performance limitations of the pumps. It must be understood that each point on a map represents a different pump. Therefore, any one curve provides an array of ideal design possibilities. This should not be confused with the performance variation due to changing operating conditions of an individual pump.

Each map shows that as  $T_{t1}/T_{t0}$  is increased along lines of constant  $P_{t1}/P_{t0}$ , the maximum mass flow ratio increases. This increase is greatest up to a temperature ratio of 2. Beyond this temperature ratio, the effect of heating is less apparent.

Figures 13 through 15 are performance maps for optimum constant area pumps. The lines of constant temperature ratio are continuous; however, there are points of discontinuity in the slope of each line. The dashed curve connecting these points divides each map into two distinct regions. The lower left area of the map corresponds to a flow limitation as shown by curve A of Figure 3. The area to the right of the dashed curve exhibits a flow cut-off similar to that illustrated by case B. These figures also show that the secondary mass flow rate

(  $\dot{m}_2$  ) decreases as the discharge total pressure ratio increases.

Appendix C contains the solutions of the computer programs for the constant area and constant pressure cases. Each line of data represents the parameters defining one optimum pump. The momentum coefficient is constant for each page of data. Area, velocity, and temperature ratios were included in these results to provide additional criteria for jet pump design. The results show that for a given  $C_M$  and  $T_{t1}/T_{t0}$ , the discharge total temperature ratio can be decreased by increasing  $P_{t1}/P_{t0}$ . This fact could be an important consideration in choosing the material for the mixing tube section.

The important effect of heating the primary air to decrease the compressor power required has been shown. It must be realized that these are idealized cases that represent the maximum performance attainable. Actual jet pumps can approach but never exceed these limits.

#### Fixed Area-Variable Discharge Pressure

The final theoretical analysis considered is to investigate the operating characteristics of jet pumps with a fixed area ratio (  $A_2/A_3$  ). By systematically varying the supply pressure and temperature ratios, it is possible to predict the envelope of operation for a given pump configuration. These predicted values are again based on ideal conditions; therefore, they represent the maximum performance attainable.

The fundamental objectives of this analysis are; first, to marry the previously discussed theory to an actual hardware design; second, to determine the design limiting parameters such as size, available compressor power and material limitations; and finally, to predict the performance curves of the jet pumps designed.



The theory of the ideal flow through a fixed area pump is similar to the analysis discussed in the constant area section. However, by fixing the area ratio ( $A_2/A_3$ ) and allowing  $P_{t5}/P_{t0}$  to vary it is possible to obtain explicit solutions for the equations of continuity, energy and momentum. Appendix C contains an example of the data obtained from the computer program used to predict each pump's performance.

The constant area system is selected primarily due to its relatively simple mixing section design. The supply temperature ratios are limited to three or less, and supply pressures are investigated up to ratios of 2.4. Mixing tube diameters of four, five, and six inches are chosen. This choice is predicated on the nominal pipe sizes available. The primary jet nozzles are designed to give a good spread of area ratios ( $A_2/A_3$ ). The final result is nine different jet pump configurations. The table below shows the area ratios of the jet pumps investigated.

		NOZZLE EXIT DIAMETER		
		2.00"	2.25"	2.60"
MIXING TUBE DIAMETER	4"	3.000	2.160	1.367
	5"	5.250	3.938	2.698
	6"	8.000	6.111	4.325

Table 1. Area Ratios Selected For Proposed Design

Figures 16 through 24 show the predicted operating maps for the nine

fixed area pumps considered. These curves are prepared primarily for a quick comparison of theoretical and experimental results. Consider the pump with an area ratio of 2.16, shown on Figure 16. Assume that  $T_{t1}/T_{t0} = 1.0$ ,  $P_{t1}/P_{t0} = 1.6$  and the back pressure ratio ( $P_{t5}/P_{t0}$ ) is adjusted to 1.2. Entering the map with these values predicts a mass flow ratio of 1.07. Now if the primary fluid is heated to  $T_{t1}/T_{t0} = 3.0$ , it can be seen that the mass flow ratio increases to 1.33. This is another example of how heating favorably influences the performance of a jet pump.

It must be clearly understood that these mass flow ratios are simply upper performance limits. Actual jet pumps will not reach these values due to friction and secondary mixing effects.

#### Theoretical conclusions

For fixed input and output conditions, all performance parameters vary favorably with an increasing mass flow ratio.

For fixed supply and demand conditions, there is one and only one area ratio,  $A_2/A_3$ , that will produce the maximum possible mass flow ratio and hence the best all around performance.

Heating the primary air decreases the compressor power required, i.e., there is a trade-off between heat and power.

The optimum pumps are idealized cases that represent the maximum performance attainable. Actual jet pumps can approach but never exceed these limits.

The ordinary ideas of efficiency are not necessarily decisive in the application of this device to boundary layer control.

### 3. Jet Pump Test Facility Design

The test facility proposed for studying heated jet pump operation is shown in Figure 25. Basically, it consists of (1) a primary flow system for supplying jet air at a desired pressure and temperature; (2) a secondary air supply system; (3) a set of interchangeable converging jet nozzles; and (4) the constant area mixing tubes.

The test apparatus was designed to be a fixed installation exhausted to the atmosphere. The primary purpose of this facility is to provide fundamental information on jet pump design parameters, and to establish correlation between experimental and theoretical data.

#### Primary Air Supply System

The primary flow section includes the compressed air supply and fuel supply systems and the combustion chamber assembly. The purpose of the primary system is to provide heated compressed air to the jet pump.

The flow of compressed air to the jet pump is controlled by means of an electrically actuated butterfly valve. It may assume any position between fully closed and fully opened. Thus, this valve controls the pressure of the primary fluid downstream.

After passing through the throttle valve, the compressed air flows into the primary system plenum chamber. The dimensions of this reservoir are arbitrary; however, the chamber must be large enough to serve as a settling tank.

The compressed air leaves the plenum chamber and passes through a square-edged orifice installed in the four inch mild steel exit pipe.

This orifice measures the flow rate of the primary air,  $\dot{m}_3$ . The compressed air is then directed into the burner assembly.

The burner unit is a small cannular combustion chamber obtained from an auxiliary gas turbine compressor used for starting jet aircraft engines. The fuel nozzle, igniter plugs and the flame tube are integral parts of the burner unit. The operating range imposed on the jet pump (i.e.,  $P_{t1}/P_{t0} \leq 2.4$  and  $T_{t1}/T_{t0} \leq 3.0$ ) is well within the capabilities of the combustion chamber.

Fuel is supplied to the burner from a pressurized fuel tank. The fuel pressure is maintained constant by means of a pressurized bottle of inert gas. Fuel flow is manually controlled by a needle valve installed in the fuel supply line. This valve sets the fuel/air ratio and is the primary means of controlling the total supply temperature,  $T_{t1}$ . The ignition system consists simply of two 12 volt storage batteries connected in series and an ignition control switch.

After the primary air is heated, it passes through a 17 inch section of 5 inch stainless steel pipe. This pipe section is inserted into the secondary air plenum chamber and provides a male fitting for the primary jet nozzle. Total supply temperature and pressure are measured by means of a fixed Kiel-temperature probe. These two values combined with the flow rate are the parameters of prime importance in the primary supply system.

The primary air then passes through a converging nozzle and is ejected into the mixing tube section. The three interchangeable nozzles designed are shown on Figures 26 through 28. These stainless steel nozzles must be machined with close dimensional control and polished to

a mirror finish. They are tapered to ensure uniform flow of primary and secondary air at the mixing tube entrance. A lubricant, such as "silver goop," should be applied to the nozzle threads prior to installation. This material prevents "welding" at elevated temperatures.

### Secondary Air System

The secondary air system simply provides a means to measure the secondary air flow rate,  $\dot{m}_2$ . Basically, it consists of an airtight plenum chamber and the associated piping and fittings for a mass flow determination.

Figure 29 shows the proposed secondary air plenum chamber. The reservoir is constructed of 12 gage sheet steel. Standard flanges are used for all piping connected to the chamber. Care must be exercised during fabrication to ensure the airtight integrity of the plenum. Any leaks in the plenum will render the secondary air mass flow measurement inaccurate.

The alignment of the primary air supply duct and the mixing tube section is important. In order to achieve the desired results it is necessary that the jet nozzle and mixing tube centerlines coincide.

### Mixing tube

The length of the mixing tube must be such that optimum mixing is accomplished. Three tubes are available with diameters of four, five, and six inches. Each tube consists of four, 24 inch pipe sections which provide for length to diameter (L/D) ratios of 4.8 to 24. L is the length of the mixing tube downstream of the nozzle exit.

The maximum wall temperature attainable is 1100°F; therefore, a heat resistant alloy is advisable. Type 304, schedule 40 stainless



steel which has a melting range of 2550-2650°F and a scaling temperature of 1550°F was selected as a suitable material. With careful attention to the discharge total pressure for fixed supply pressure and temperature ratios, it would be possible to hold the temperature in the mixing tube low enough so that a mild steel or aluminum could be used. This is considered to be an additional experimental burden that can be alleviated by using stainless steel throughout.

One of the simplifying assumptions was to neglect wall friction; therefore, the internal surface of the mixing tube is to be polished to a smooth surface to minimize frictional losses. Consequently, there should be closer agreement between theory and experiment.

By taking temperature and pressure readings along the mixing tube, it is possible to determine where optimum mixing is achieved. Any additional length will only cause a net additional loss in head due to friction. Many of the design questions concerning optimum length of the mixing section can be answered only by experimentation and by investigation of the kinetics of the mixing process in which all effects are considered.

As shown in Figure 27 an oversized flange connects the mixing tube to the secondary air plenum chamber. Attached to this flange is a bell-mouth annulus machined from hot rolled steel. The converging secondary nozzle is formed by this annulus and the exterior of the primary jet nozzle.

The three mixing tubes can be interchanged and may be used with each of the primary nozzles. This flexibility provides for experimental tests of nine different jet pump configurations.

In an actual application, a diffuser would, of course, be used at

the end of the mixing tube to bring the mixed fluids gradually to atmospheric pressure. For the purposes of the present research, separate diffusers were not designed due to the many varying operating conditions and physical configurations.

The mixing tube is supported by three stanchions similar to the one illustrated in Figure 30. Each stanchion has two turnbuckles which allow alignment in the vertical and horizontal planes.

The handling cart shown in Figure 31 will be used whenever it is necessary to change the mixing tubes. Two carts of this design would be desirable to provide the versatility prescribed for the test facility. The carts are made almost exclusively of aluminum channel and angle stock. The maximum design load is 1000 lbs.

#### Instrumentation

A typical 24 inch mixing section with the associated instrumentation is shown in Figure 32. Each section contains six static pressure taps. These taps are located at four inch intervals along the entire mixing tube. Since the assumption is made that a constant static pressure exists across each cross section, the static pressure is measured only at the mixing tube walls.

Total pressure and total temperature readings are to be taken with a Kiel-temperature probe. These readings can be taken at any of eight different stations along the mixing tube. At each station there are two probe fittings which permit transverse surveys of temperature and pressure along either of two diameters.

The mounting plates shown at each station support the receiver containing the Kiel-temperature probe. The receiver is mechanically



controlled remotely by a transmitter. This transmitter controls the transverse and angular movement of the probe.

When the probe is inserted in one of the probe fittings, the remaining holes are plugged. These plugs are machined to be flush with the inside walls of the mixing tube.

A butterfly valve is installed in a pipe section after the mixing tube to regulate the discharge total pressure. A second Kiel-temperature probe is available to be installed at the end of the mixing tube to measure this pressure.

This test facility provides the means to determine the primary and secondary mass flow rates. In addition, velocity and temperature profiles may be determined along the length of the mixing tube, thereby enabling determination of an optimum mixing length.

#### Mass flow measurement

It has been shown that one of the most important performance parameters is the mass flow ratio,  $\dot{m}_2/\dot{m}_3$ . Figure 33 shows the installation of two square-edged orifices. The orifice plates for the primary and ambient air supply have holes of 2.5 and 3.75 inch diameter respectively. They are installed in slip-on, butt welded flanges which have pressure taps meeting ASME specifications. The two flange taps are connected to a water filled manometer board where the pressure difference,  $h_w$ , is measured. The gage pressure  $P_1$ , and temperature  $t_1$ , must be measured ahead of each orifice.

ASME requirements prescribe a minimum straight pipe length ahead of and behind an orifice for a flow rate measurement. The straight pipe length shown in Figure 25 exceeds the minimum requirements; therefore, it is not necessary to install flow straightners.

Reference 3 contains all necessary equations for determination of the flow rates.

#### Cost estimate

The cost estimate is based on the following assumptions:

- a. No old material will be used except the burner and fuel supply system, pressure and temperature gages, and manometer boards.
- b. Heat resistant alloy is necessary in the mixing tube.
- c. All machining and fabrication can be accomplished by shop personnel.
- d. Three nozzles and three mixing tubes are necessary to check a range of configurations.

Two cost estimates are provided:

- |  |           |
|--|-----------|
| a. Total material cost with remote electrical control: | \$5555.42 |
| b. Total material cost with remote mechanical control: | \$4787.42 |

The total man-hour labor estimate is 362 hours.

The material cost could be reduced considerably if only one nozzle and one mixing tube were used. This is shown in the following estimate:

- |  |           |
|--|-----------|
| a. Total material cost with remote electrical control: | \$3734.08 |
| b. Total material cost with remote mechanical control: | \$2969.08 |

A minimum cost estimate is provided based on the following assumptions:

- a. One nozzle and one mixing tube will be used.
- b. The mixing tube consists of two, 24 inch mixing sections

instead of four.

c. Handling carts are not required.

d. The supply temperature ratio is reduced to  $T_{t1}/T_{t0} = 2.0$ .

Therefore, stainless steel could be replaced by mild steel.

e. Manual controls are used instead of electrical.

f. As much old material as possible will be used such as plastic tubing, wall taps, copper tubing, connectors, pressure plugs, burner and fuel supply system pressure and temperature gages, and manometer boards.

g. Four inch pipe will be used in the mixing tube.

The total minimum material cost is \$1601.50.

An itemized cost estimate is contained in Appendix B.

#### 4. Recommendations for further study

At this stage of the investigation, the most essential requirement is to build the test facility described in this report. If this is not feasible at the present time for economic or other reasons, a further theoretical analysis would be useful to study the effects of wall friction and incomplete mixing. These analytical studies should also include the effects of variable specific heats.

A further recommendation is to proceed with the preliminary design of an experimental boundary layer control system utilizing such a heated jet pump.

It also would be of interest to investigate the performance of a heated jet pump with a converging-diverging primary nozzle.

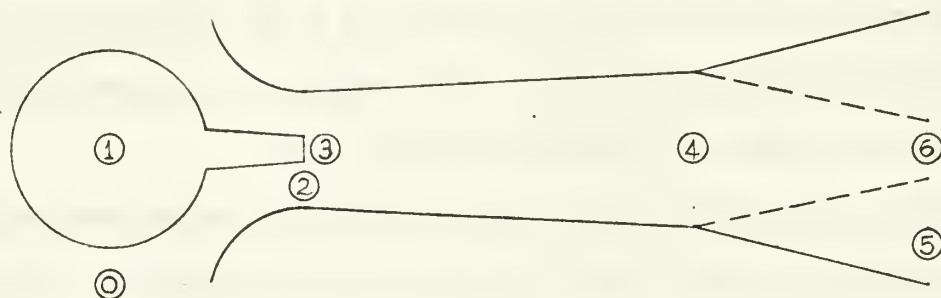
RIDDER  
SUMMERS

APRIL 1966

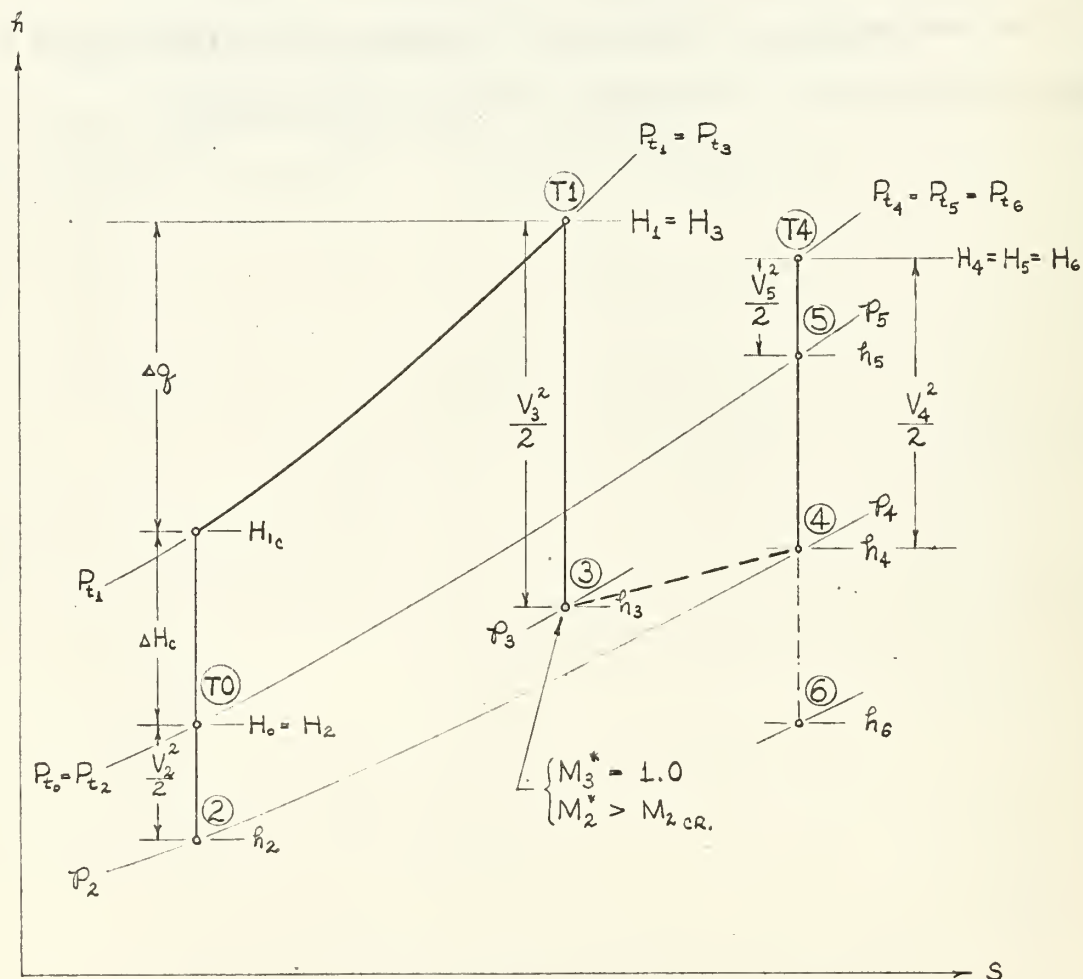
USNPGS DEPARTMENT OF AERONAUTICS

FIGURE 1

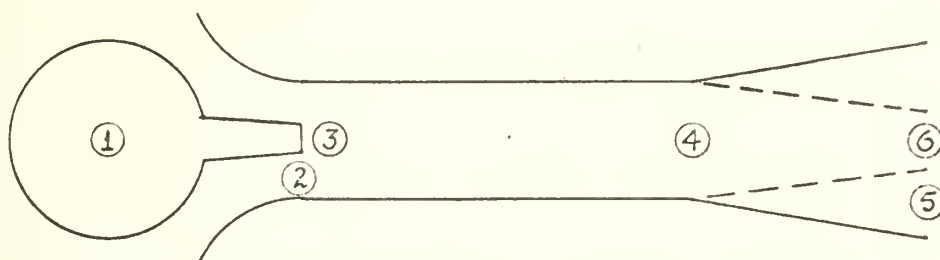
TYPICAL ENTHALPY ENTROPY DIAGRAM FOR  
A CONSTANT PRESSURE HEATED JET PUMP



SKETCH DENOTING STATION LOCATIONS



RIDDER SUMMERS APRIL 1966	USNPGS DEPARTMENT OF AERONAUTICS	FIGURE 2
	TYPICAL ENTHALPY-ENTROPY DIAGRAM FOR	
	A CONSTANT AREA HEATED JET PUMP	



① SKETCH DENOTING STATION LOCATIONS

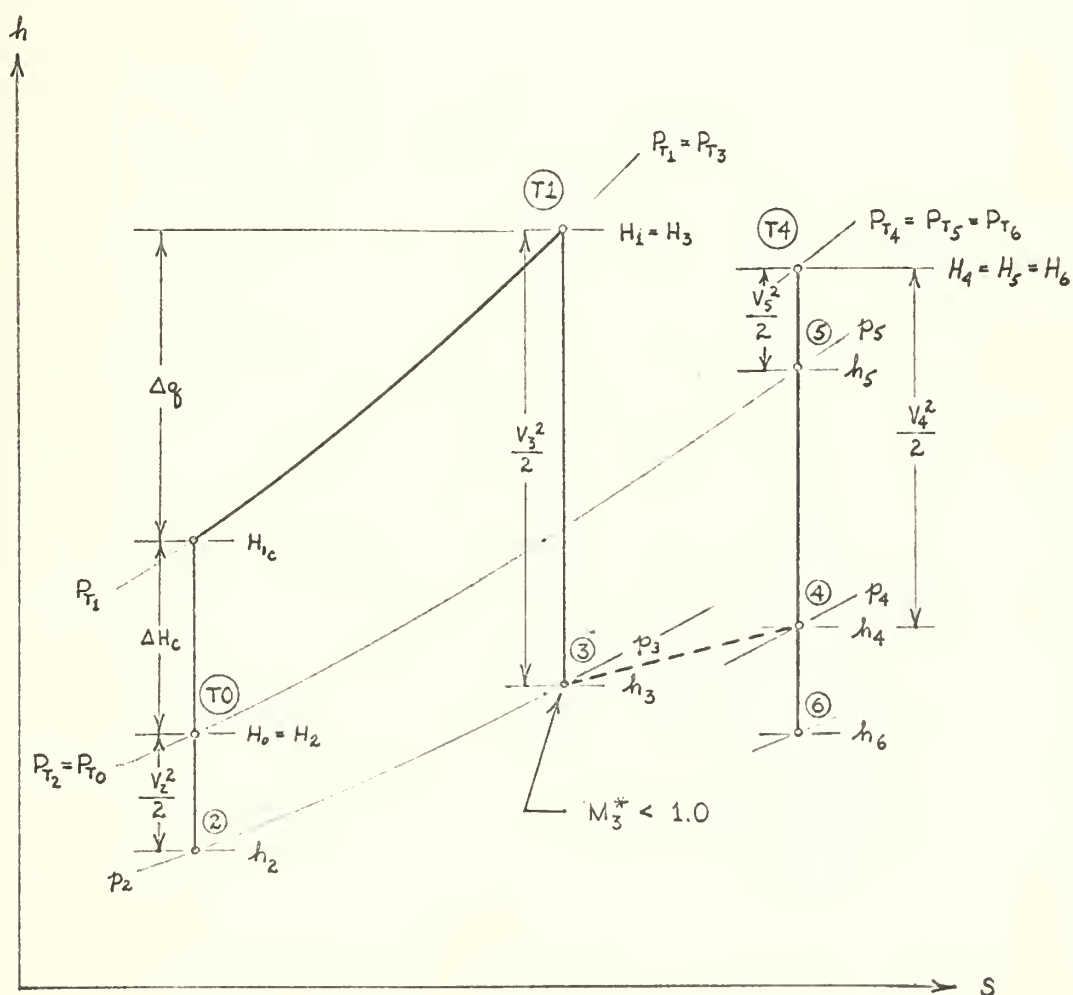




FIGURE 3  
PREDICTED PERFORMANCE CURVES FOR  
TWO FAMILIES OF CONSTANT AREA JET  
PUMPS, ILLUSTRATING THE TWO OPTIMUM  
CONDITIONS FOR THE CONSTANT AREA CASE.

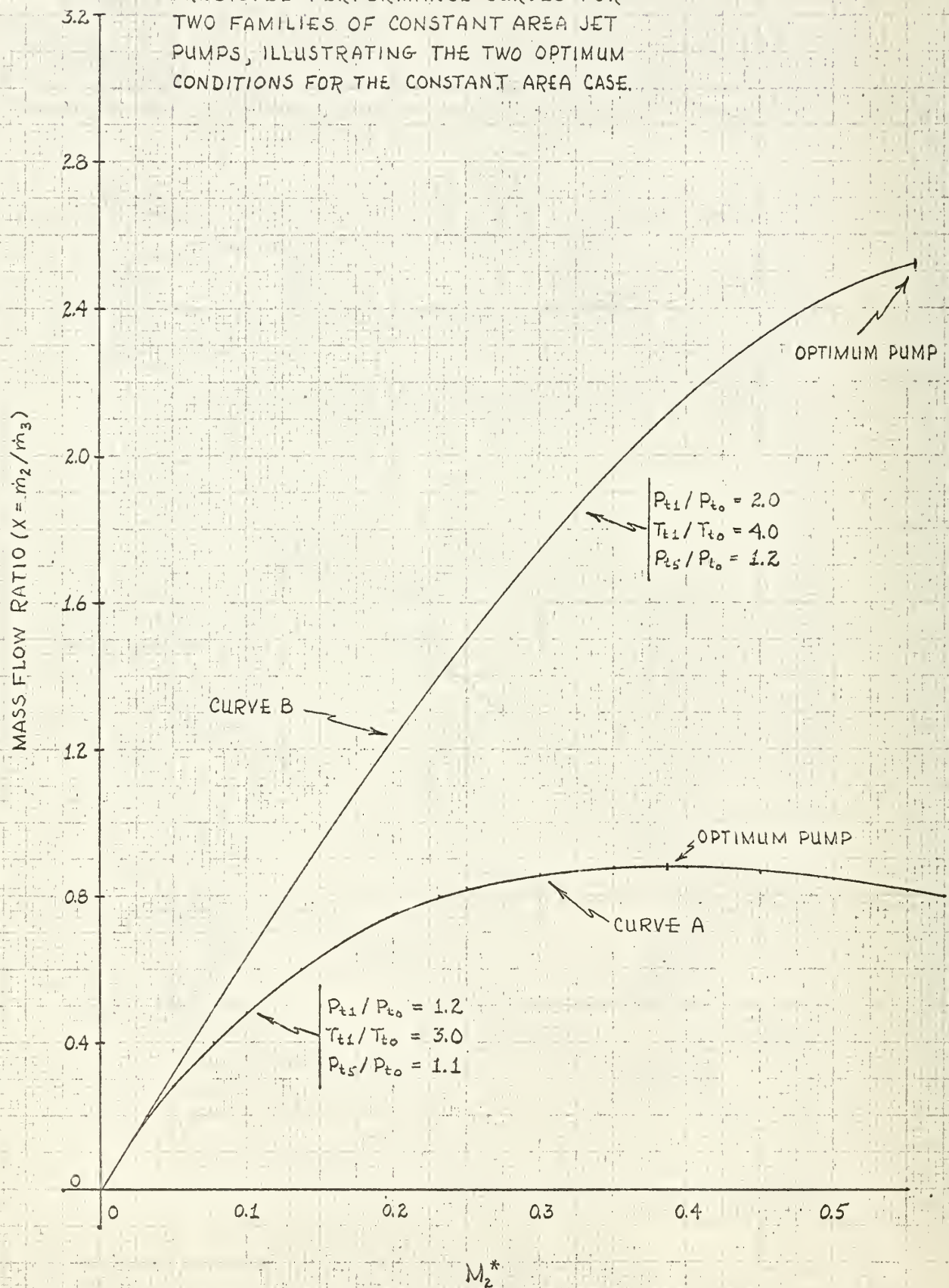




FIGURE 4

THEORETICAL MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS, SHOWING COMBINATIONS OF  $C_{PM}$  AND  $C_{QM}$  WHICH SATISFY THE CONDITIONS:

$$P_{t5}/P_{t0} = 1.100$$

$$C_m = 0.442$$

NOTE: Dashed lines denote constant pressure ratios ( $P_{t1}/P_{t0}$ ), and solid lines show curves of constant temperature ratios

HEAT-MOMENTUM COEFFICIENT ( $C_{QM}$ )

POWER-MOMENTUM COEFFICIENT ( $C_{PM}$ )

$$\frac{P_{t1}}{P_{t0}}$$

$$\frac{T_{t1}}{T_{t0}}$$

1.6

2.0

2.4

2.8

5.0

4.0

3.0

2.0

1.0

16

14

12

10

8

6

4

2

0

-1

0

.2

.4

.6

.8

1.0

1.2

FIGURE 5

THEORETICAL MAPS OF OPTIMUM CONSTANT PRESSURE JET PUMPS, SHOWING COMBINATIONS OF  $C_{PM}$  AND  $C_{QM}$  WHICH SATISFY THE CONDITIONS:

$$P_{t5}/P_{t0} = 1.200$$

$$C_M = 0.662$$

NOTE: Dashed lines denote constant pressure ratios ( $P_{t1}/P_{t0}$ ), and solid lines show curves of constant temperature ratio.

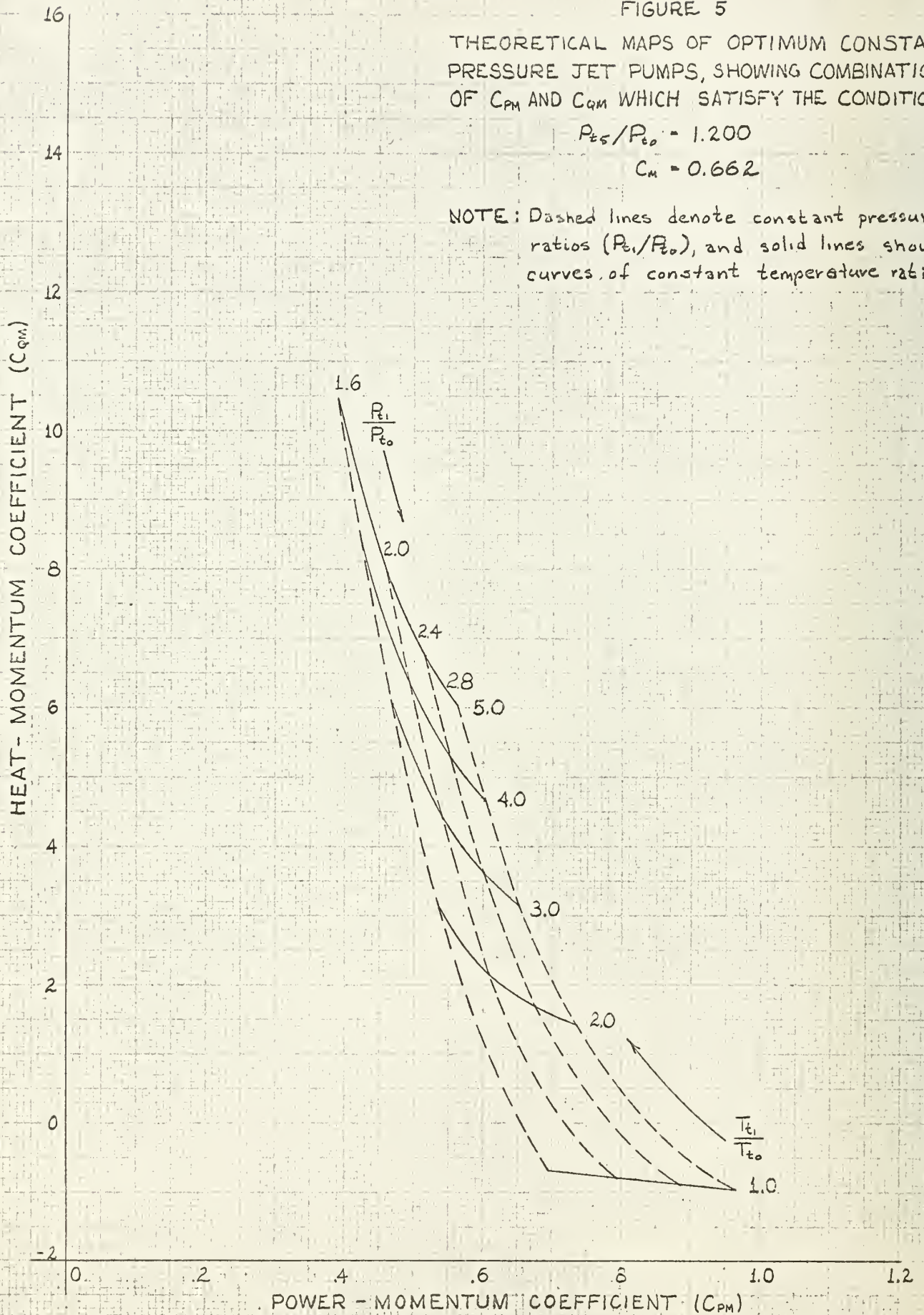


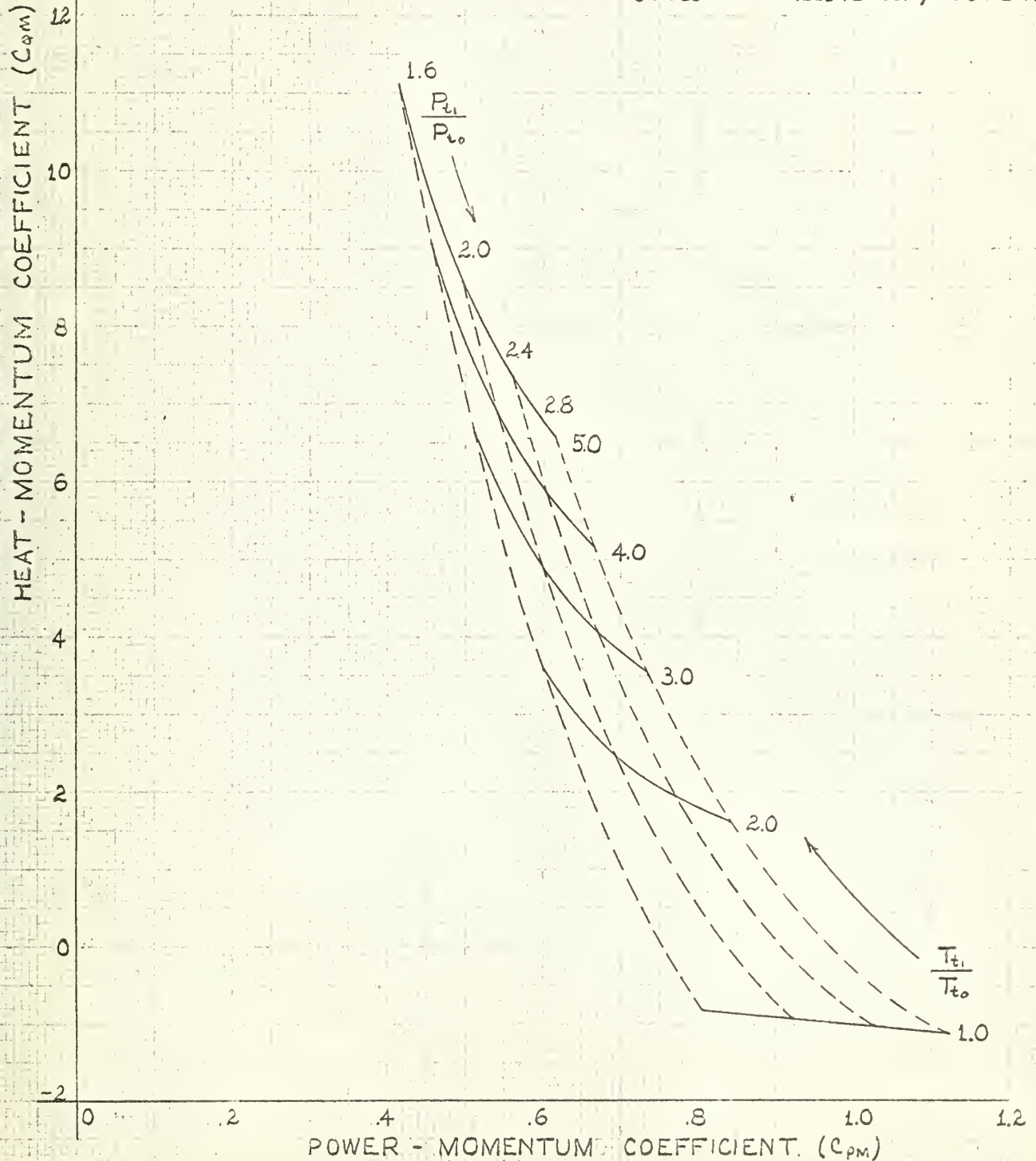
FIGURE 6

THEORETICAL MAPS OF OPTIMUM CONSTANT  
PRESSURE JET PUMPS, SHOWING COMBINATIONS  
OF  $C_{PM}$  AND  $C_{QM}$  WHICH SATISFY THE CONDITIONS:

$$P_{t5}/P_{t0} = 1.300$$

$$C_m = 0.856$$

NOTE: Dashed lines denote constant pressure  
ratio ( $P_{t1}/P_{t0}$ ), and solid lines show  
curves of constant temperature ratio.





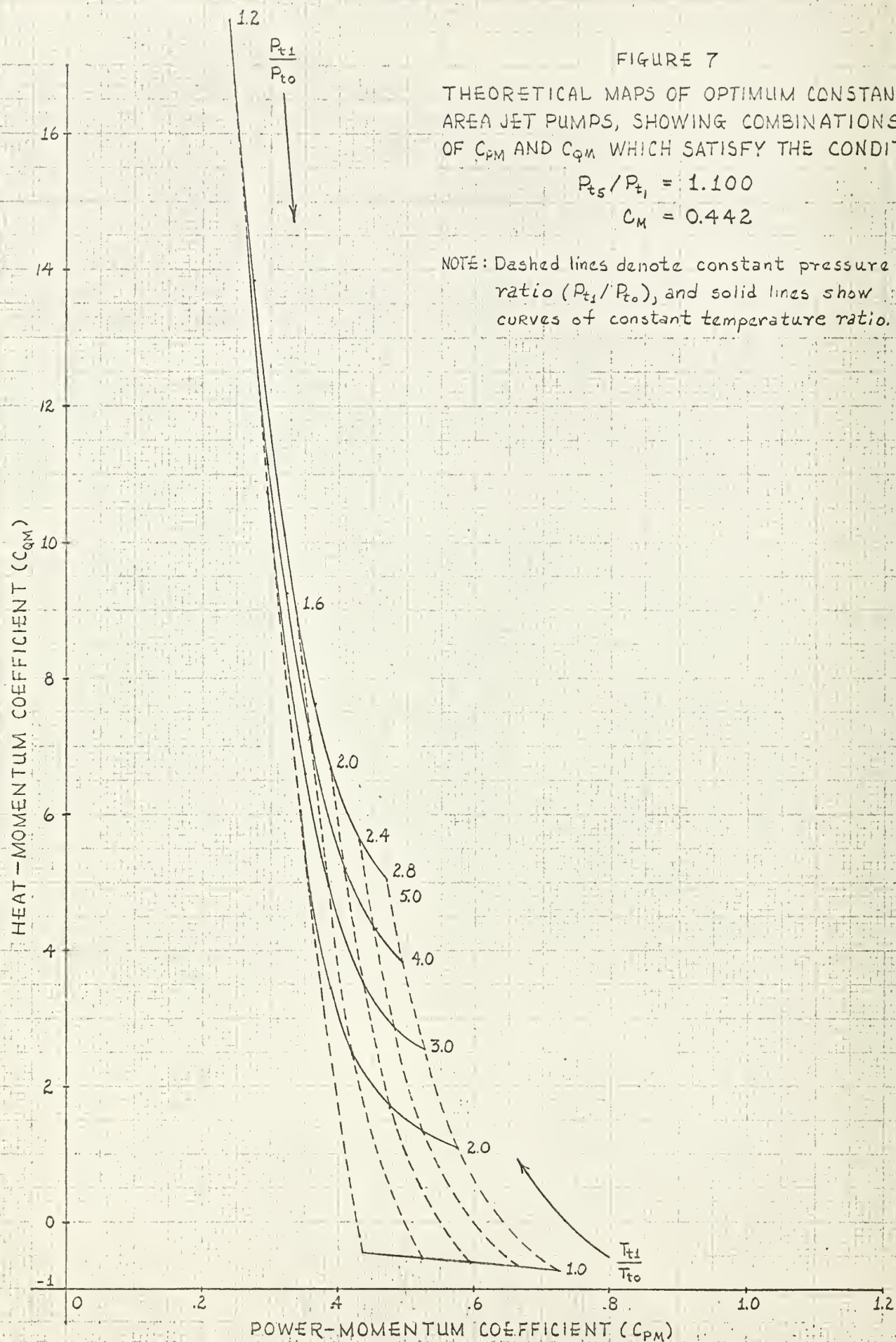


FIGURE 8

THEORETICAL MAPS OF OPTIMUM CONSTANT AREA JET PUMPS, SHOWING COMBINATIONS OF  $C_{PM}$  AND  $C_{QM}$  WHICH SATISFY THE CONDITIONS:

$$P_{t5} / P_{t0} = 1.200$$

$$C_H = 0.662$$

NOTE: Dashed lines denote constant pressure ratio ( $P_{t1} / P_{t0}$ ), and solid lines show curves of constant temperature ratio.

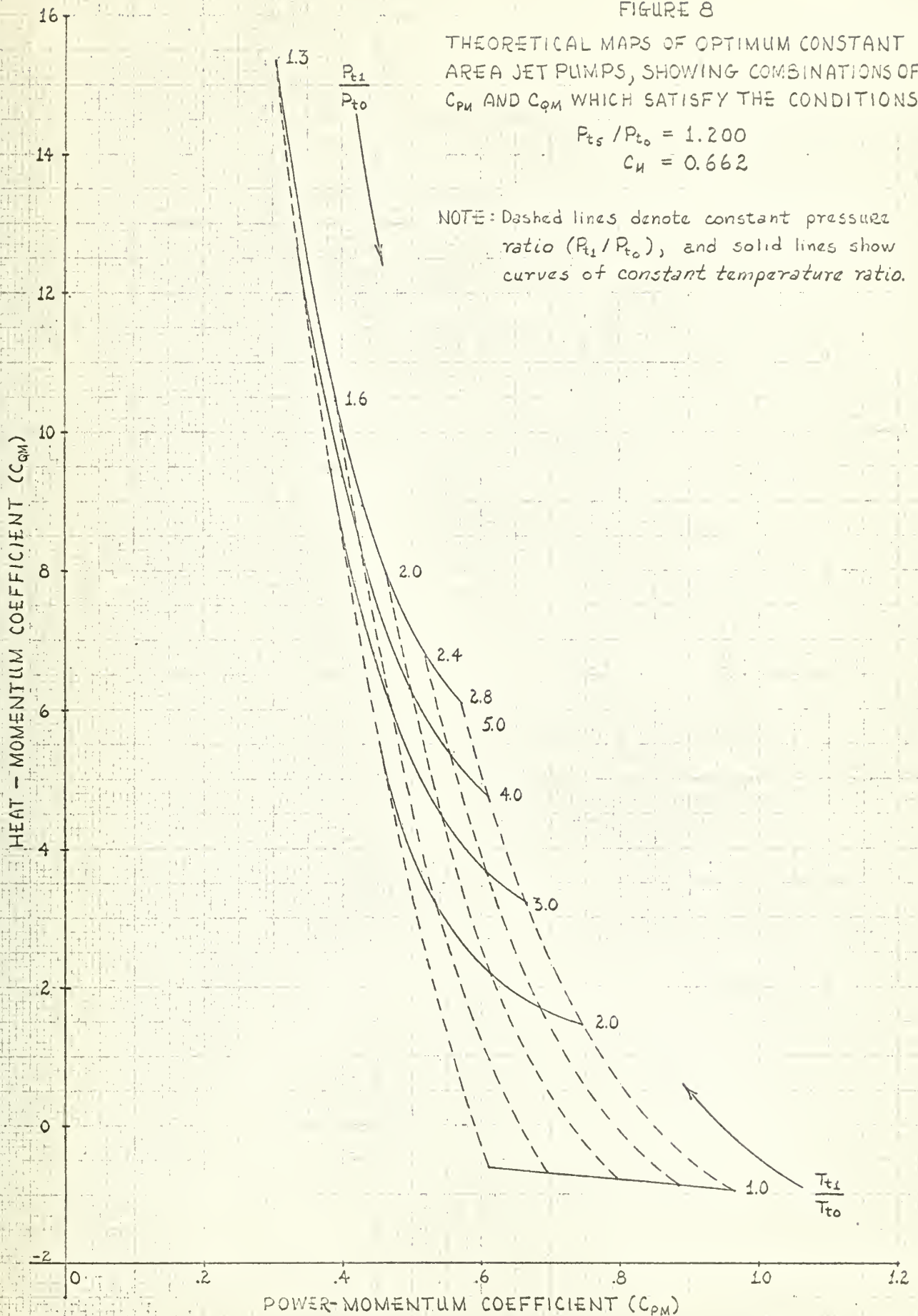




FIGURE 9

THEORETICAL MAPS OF OPTIMUM CONSTANT AREA JET PUMPS, SHOWING COMBINATIONS OF  $C_{PM}$  AND  $C_{QM}$  WHICH SATISFY THE CONDITIONS:

$$P_{t5}/P_{t0} = 1.300$$

$$C_M = 0.856$$

NOTE: Dashed lines denote constant pressure ratio ( $P_{t5}/P_{t0}$ ), and solid lines show curves of constant temperature ratio.

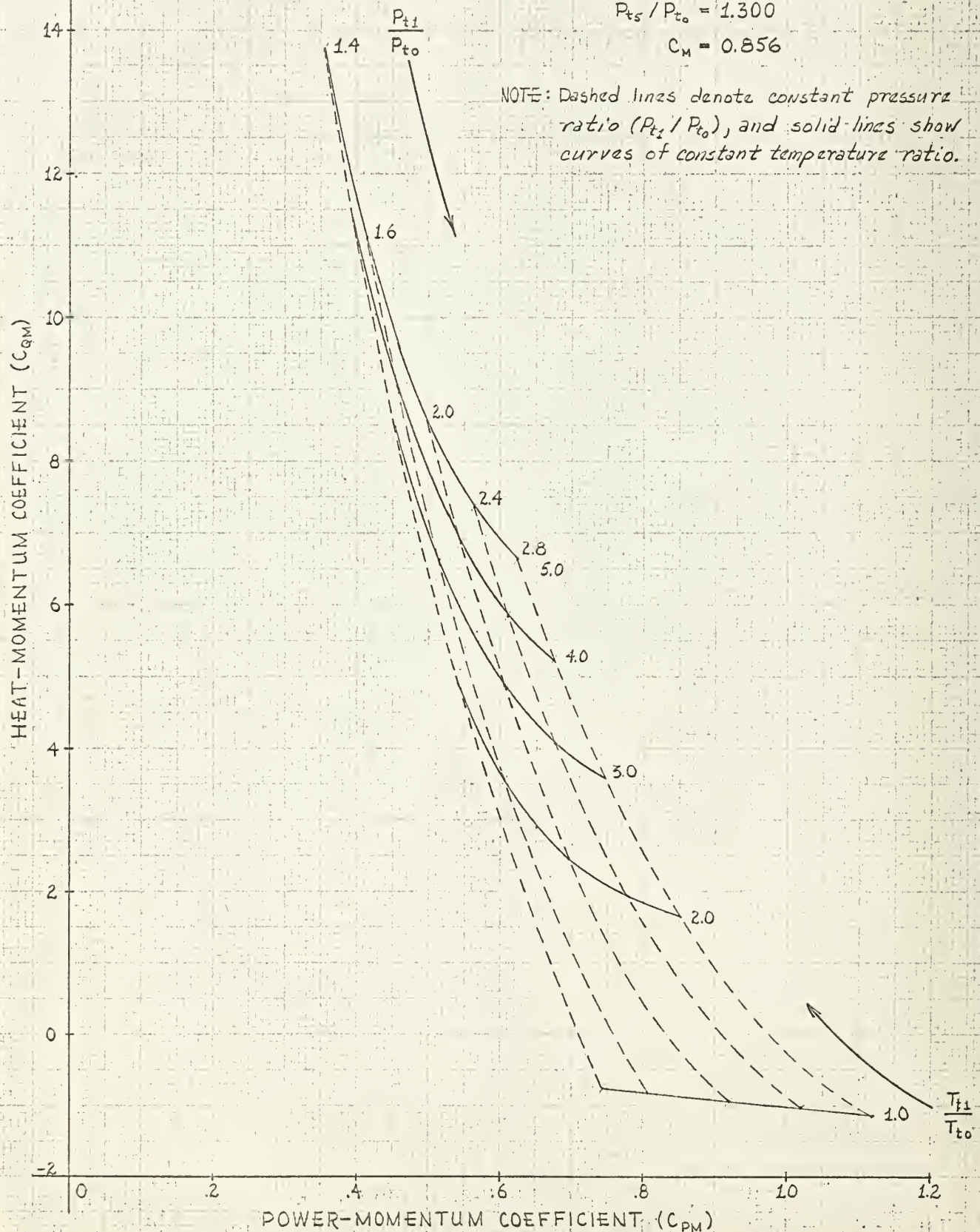




FIGURE 10  
PERFORMANCE MAPS OF OPTIMUM CONSTANT  
PRESSURE JET PUMPS, FOR  $P_c/P_{t0} = 1.1$  AND  $C_M = 0.442$

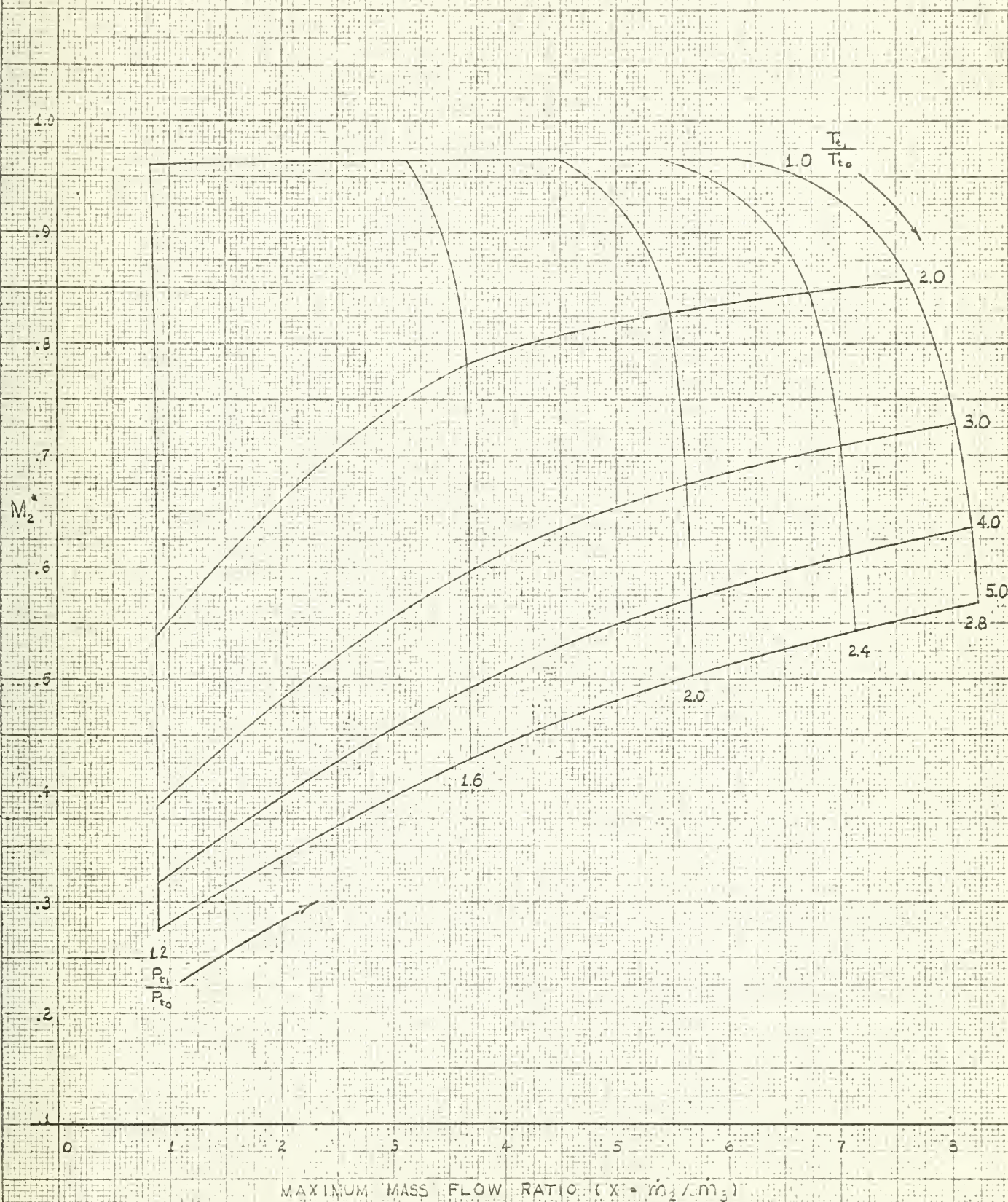




FIGURE 11  
PERFORMANCE MAPS OF OPTIMUM CONSTANT  
PRESSURE JET PUMPS FOR  $P_{t5}/P_{t0} = 1.7$  AND  $C_M = 0.662$

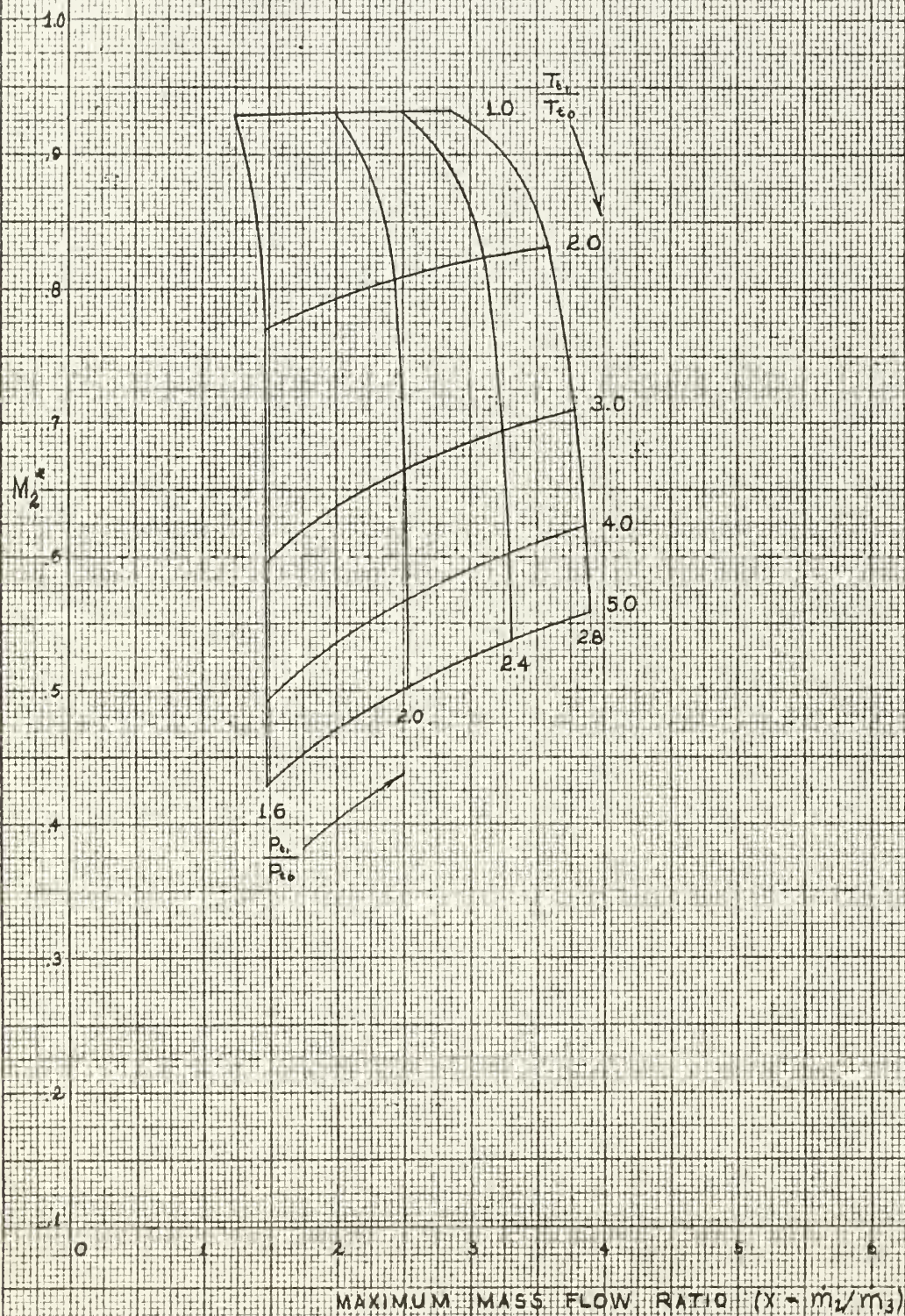




FIGURE 12  
 PERFORMANCE MAPS OF OPTIMUM CONSTANT  
 PRESSURE JET PUMPS FOR  $P_{t5}/P_{t0} = 1.3$  AND  $C_M = 0.856$

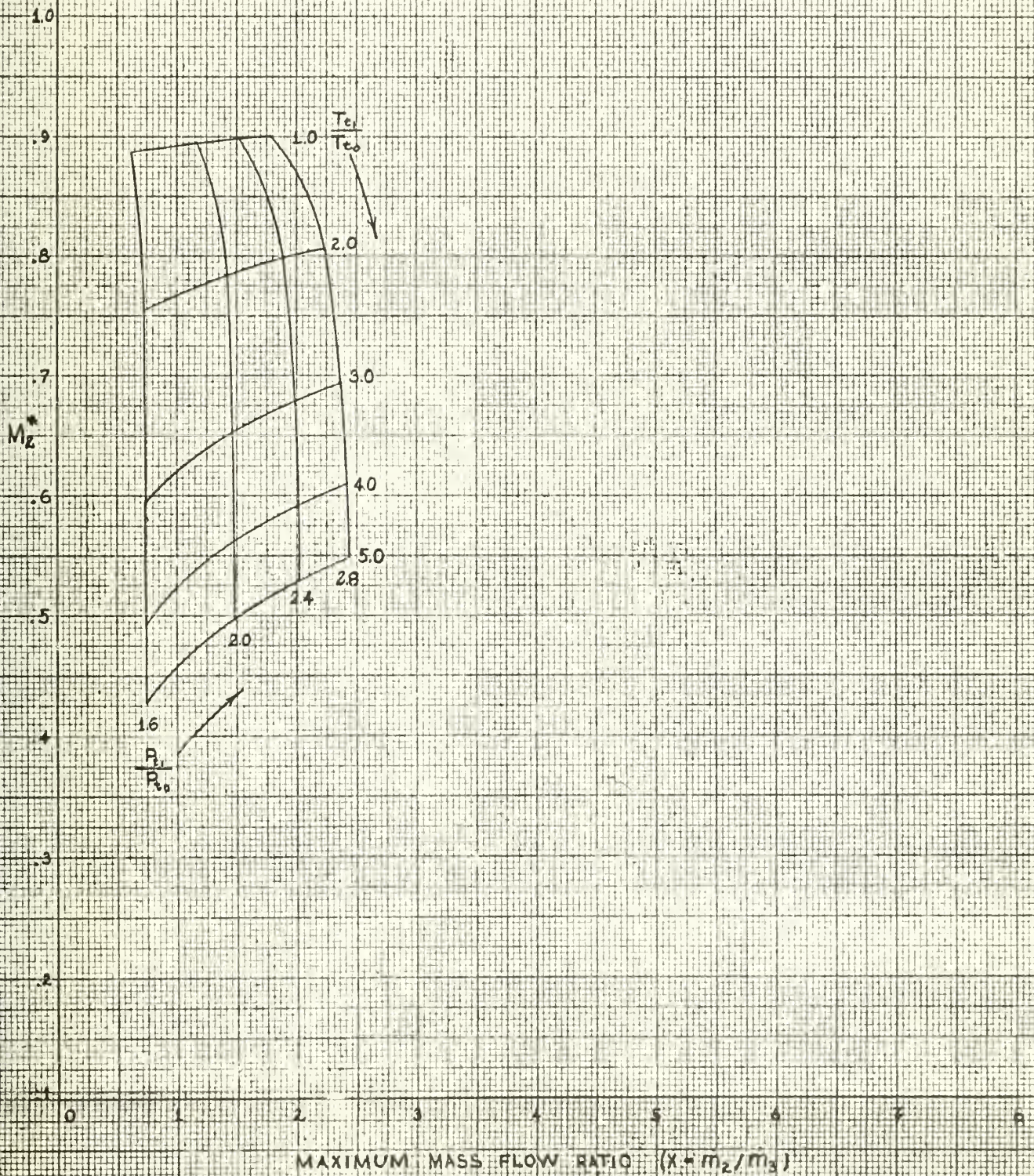




FIGURE 13  
PERFORMANCE MAPS OF OPTIMUM CONSTANT  
AREA JET PUMPS FOR  $P_{t3}/P_{t0} = 1.1$  AND  $C_u = 0.442$

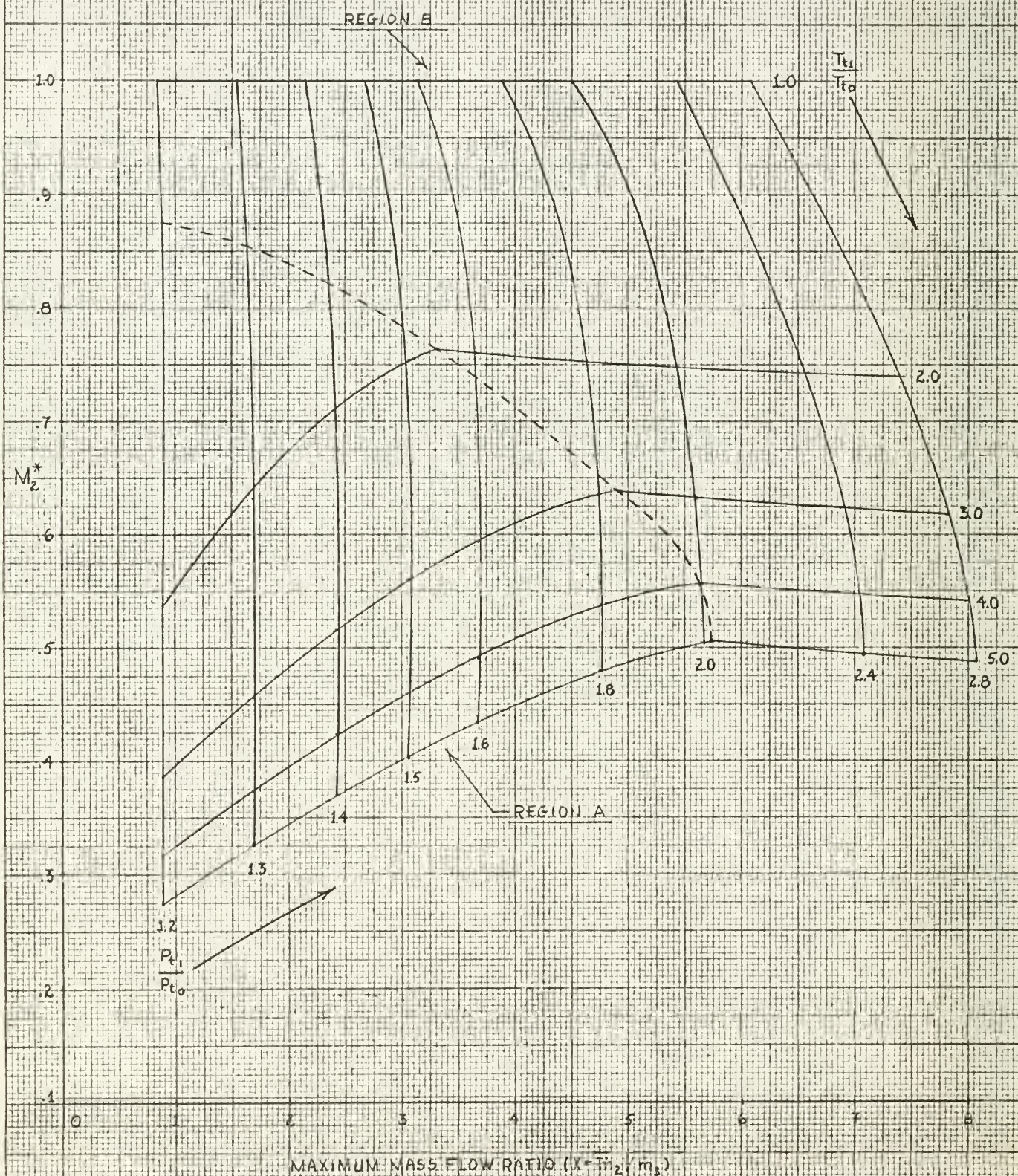




FIGURE 14  
 PERFORMANCE MAPS OF OPTIMUM CONSTANT  
 AREA JET PUMPS FOR  $P_{t3}/P_{t0} = 1.2$  AND  $C_u = 0.662$

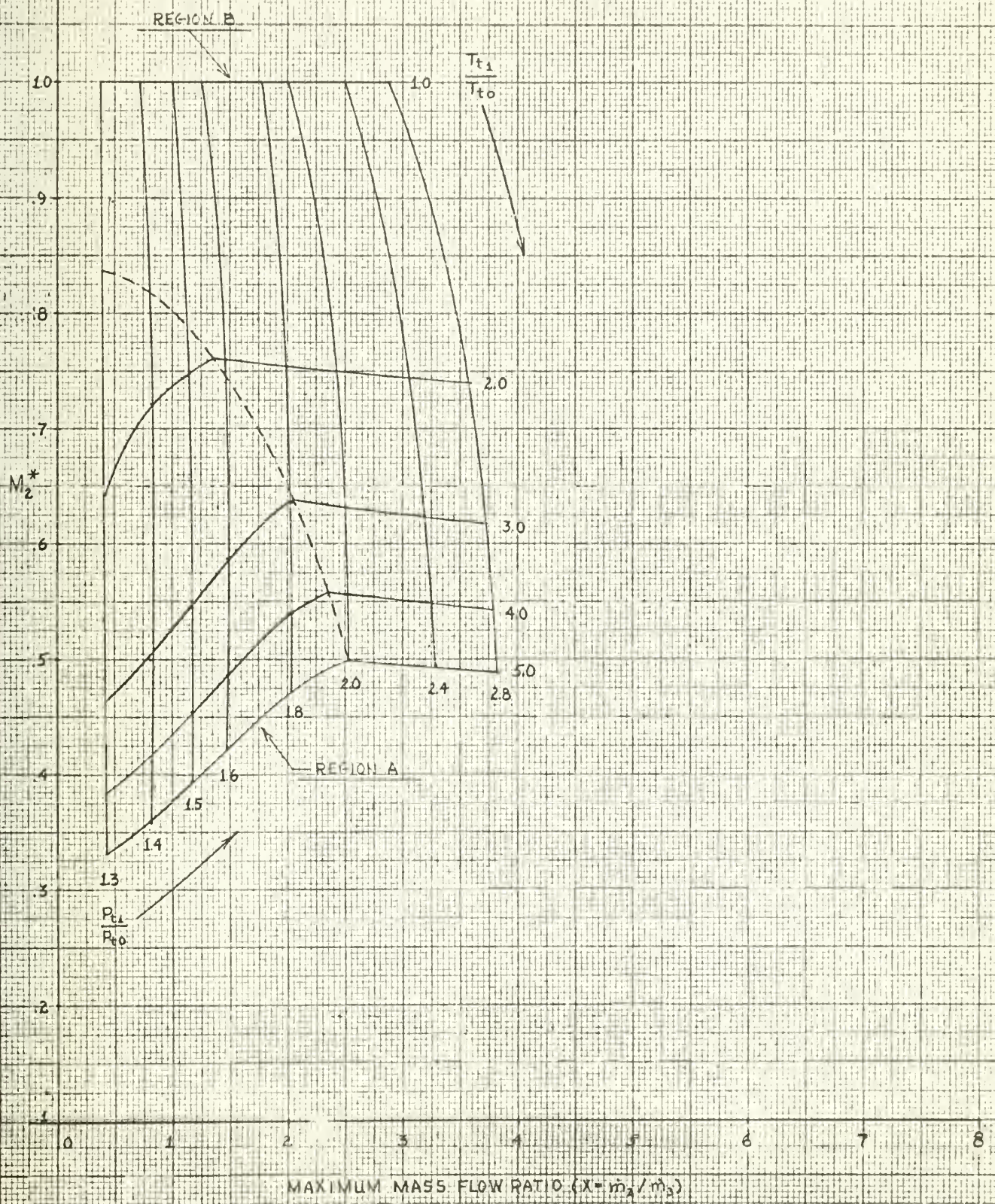




FIGURE 15  
PERFORMANCE MAPS OF OPTIMUM CONSTANT  
AREA JET PUMPS FOR  $A_3/A_0 = 1.3$  AND  $C_u = 0.856$

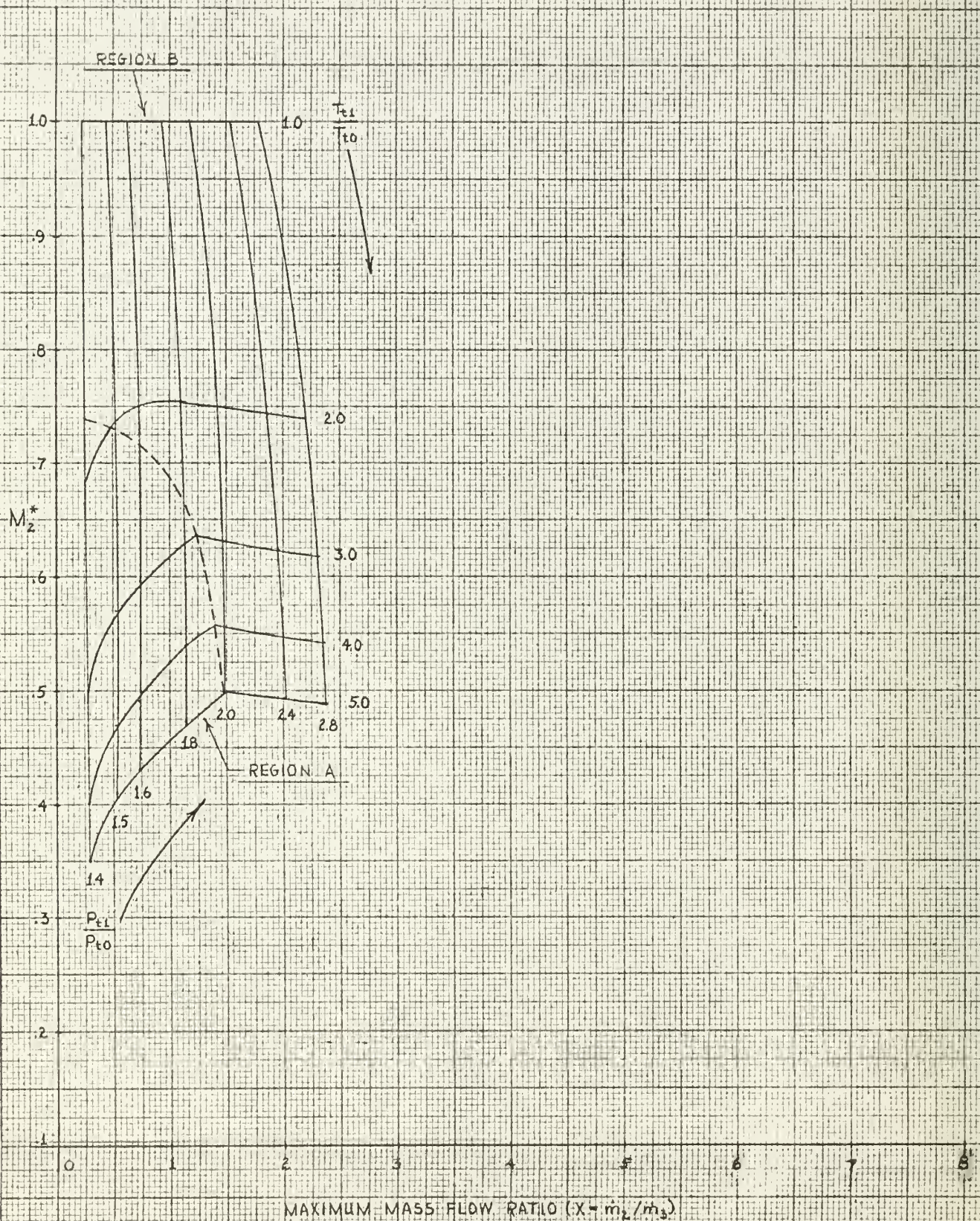




FIGURE 16  
PREDICTED MAP FOR A CONSTANT AREA  
HEATED JET PUMP WITH  $A_1/A_3 = 1.367$

CURVE 1 DENOTES  $T_{r1}/T_{r0} = 1.0$

CURVE 2 DENOTES  $T_{r1}/T_{r0} = 2.0$

CURVE 3 DENOTES  $T_{r1}/T_{r0} = 3.0$

NOTE :  $\circ \sim$  DENOTES MAXIMUM MASS  
FLOW RATIO ATTAINABLE

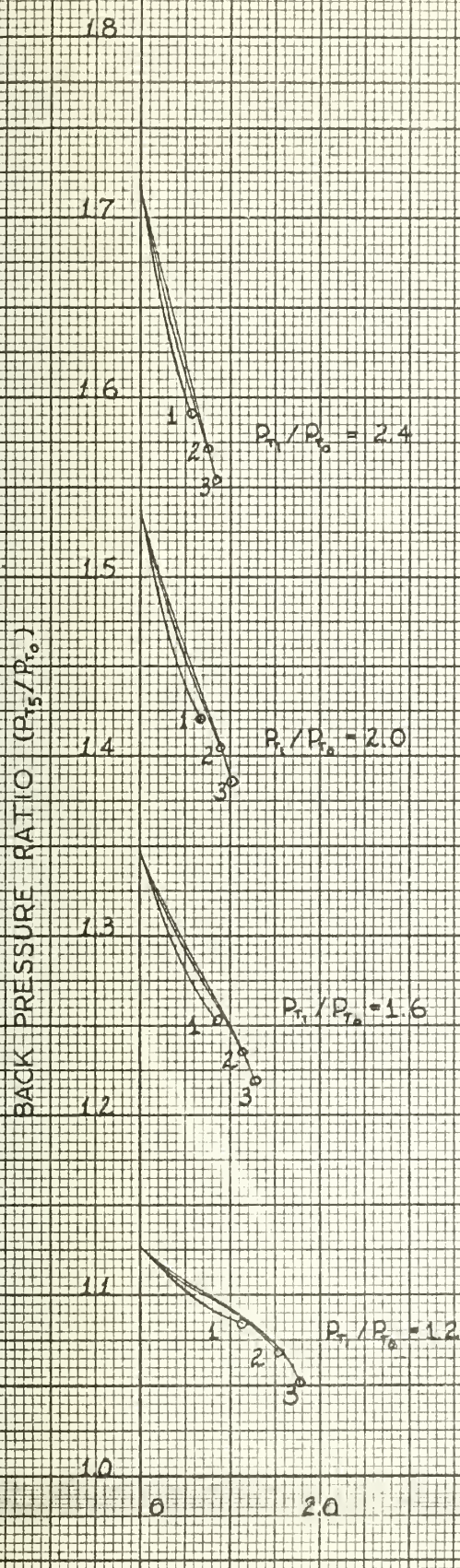




FIGURE 17  
PREDICTED MAP FOR A CONSTANT AREA  
HEATED JET PUMP WITH  $(A_2/A_3) = 2.160$ .

CURVE 1 DENOTES  $T_1/T_0 = 1.0$

CURVE 2 DENOTES  $T_1/T_0 = 2.0$

CURVE 3 DENOTES  $T_1/T_0 = 3.0$

NOTE:  $\circ \sim$  DENOTES MAXIMUM MASS  
FLOW RATIO ATTAINABLE

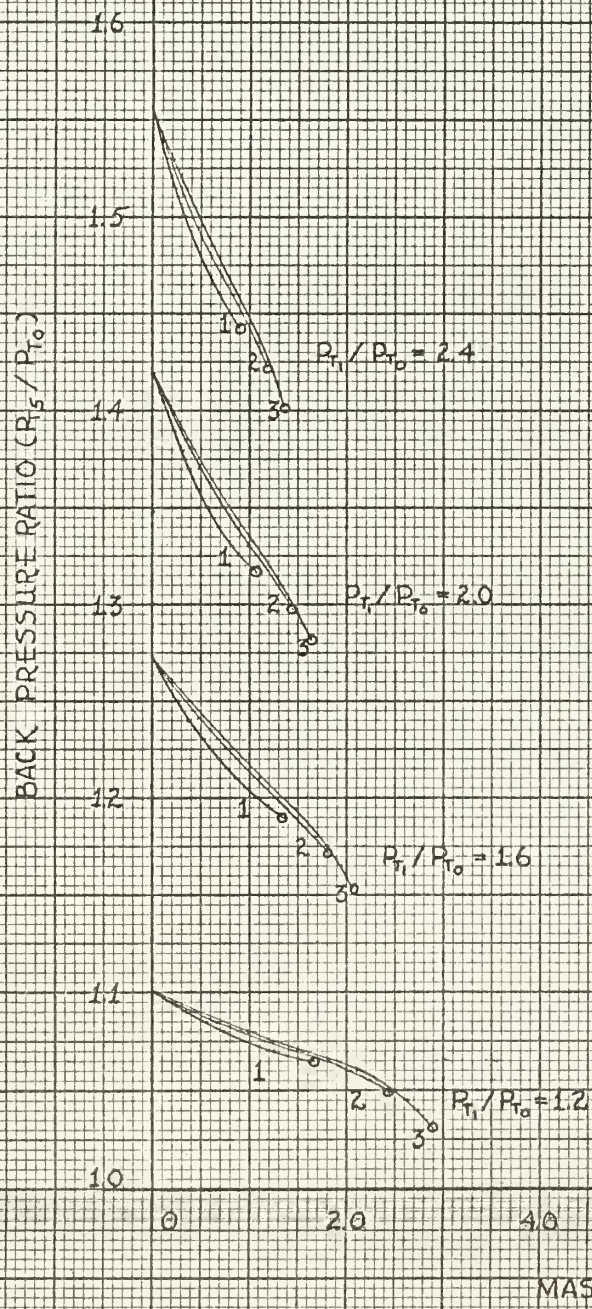




FIGURE 18  
 PREDICTED MAP FOR A CONSTANT AREA  
 HEATED JET PUMP WITH  $A_2/A_3 = 2.698$   
 CURVE 1 DENOTES  $T_1/T_0 = 1.0$   
 CURVE 2 DENOTES  $T_1/T_0 = 2.0$   
 CURVE 3 DENOTES  $T_1/T_0 = 3.0$

NOTE:  $\circ \sim$  DENOTES MAXIMUM MASS  
 FLOW RATIO ATTAINABLE

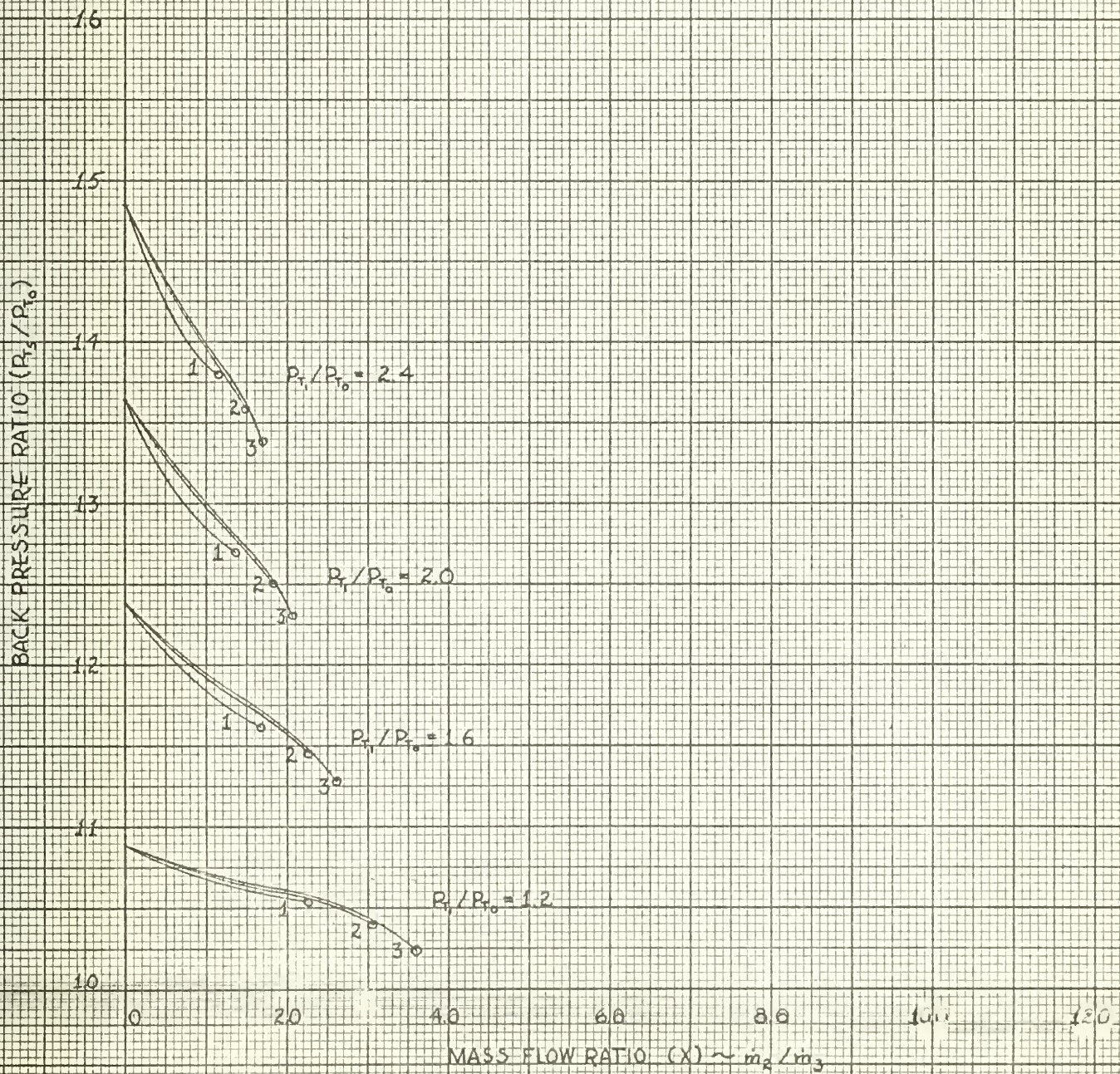




FIGURE 19  
PREDICTED MAP FOR A CONSTANT AREA  
HEATED JET PUMP WITH  $A_2/A_3 = 3.00$

CURVE 1 DENOTES  $T_{r1}/T_{r0} = 1.0$

CURVE 2 DENOTES  $T_{r1}/T_{r0} = 2.0$

CURVE 3 DENOTES  $T_{r1}/T_{r0} = 3.0$

NOTE:  $\circ \sim$  DENOTES MAXIMUM MASS  
FLOW RATIO ATTAINABLE

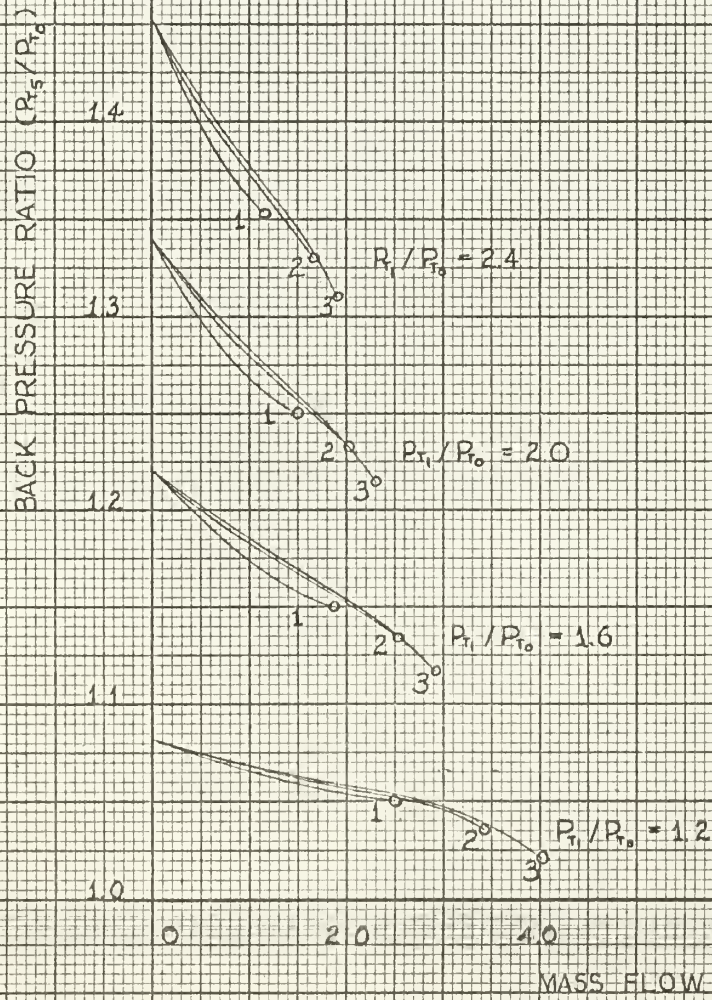




FIGURE 20  
 PREDICTED MAP FOR A CONSTANT AREA  
 HEATED JET PUMP WITH  $A_2/A_3 = 3.938$   
 CURVE 1 DENOTES  $T_1/T_0 = 1.0$   
 CURVE 2 DENOTES  $T_1/T_0 = 2.0$   
 CURVE 3 DENOTES  $T_1/T_0 = 3.0$

NOTE: o~ DENOTES MAXIMUM MASS  
 FLOW RATIO ATTAINABLE

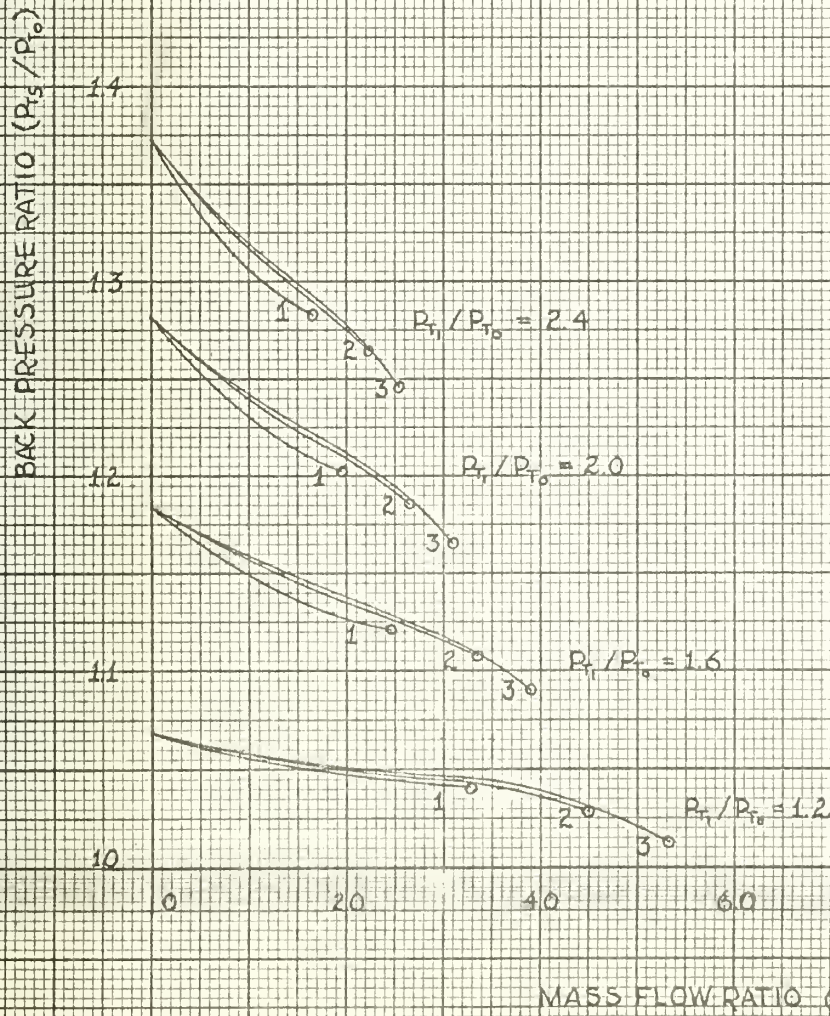




FIGURE 21  
 PREDICTED MAP FOR A CONSTANT AREA  
 HEATED JET PUMP WITH  $A_2/A_1 = 4.325$   
 CURVE 1 DENOTES  $T_h/T_c = 1.0$   
 CURVE 2 DENOTES  $T_h/T_c = 2.0$   
 CURVE 3 DENOTES  $T_h/T_c = 3.0$

NOTE:  $\circ \sim$  DENOTES MAXIMUM MASS  
 FLOW RATIO ATTAINABLE

BACK PRESSURE RATIO ( $P_2/P_0$ )

18

17

16

15

14

13

12

11

10

$P_1/P_0 = 2.4$

$P_1/P_0 = 2.0$

$P_1/P_0 = 1.6$

$P_1/P_0 = 1.2$

0

20

40

60

80

100

120

MASS FLOW RATIO (X)  $\sim m_2/m_1$



FIGURE 22  
 PREDICTED MAP FOR A CONSTANT AREA  
 HEATED JET PUMP WITH  $(A_2/A_3) = 5.250$   
 CURVE 1 DENOTES  $T_1/T_0 = 1.0$   
 CURVE 2 DENOTES  $T_1/T_0 = 2.0$   
 CURVE 3 DENOTES  $T_1/T_0 = 3.0$

NOTE:  $\circ \sim$  DENOTES MAXIMUM MASS  
 FLOW RATIO ATTAINABLE

BACK PRESSURE RATIO ( $P_{T5}/P_{T0}$ )

1.8

1.7

1.6

1.5

1.4

1.3

1.2

1.1

1.0

$P_1/P_{T0} = 2.4$

$P_1/P_{T0} = 2.0$

$P_1/P_{T0} = 1.6$

$P_1/P_{T0} = 1.2$

MASS FLOW RATIO ( $X \sim \dot{m}_2/\dot{m}_3$ )

0

2.0

4.0

6.0

8.0

10.0

12.0



FIGURE 2.3  
 PREDICTED MAP FOR A CONSTANT AREA  
 HEATED JET PUMP WITH  $(A_2/A_3) = 6.111$   
 CURVE 1 DENOTES  $T_{r1}/T_{r0} = 1.0$   
 CURVE 2 DENOTES  $T_{r1}/T_{r0} = 2.0$   
 CURVE 3 DENOTES  $T_{r1}/T_{r0} = 3.0$

NOTE: o ~ DENOTES MAXIMUM MASS  
 FLOW RATIO ATTAINABLE

BACK PRESSURE RATIO ( $P_{r3}/P_{r0}$ )

1.8

1.7

1.6

1.5

1.4

1.3

1.2

1.1

1.0

$P_{r1}/P_{r0} = 2.4$

$P_{r1}/P_{r0} = 2.0$

$P_{r1}/P_{r0} = 1.6$

$P_{r1}/P_{r0} = 1.2$

0

20

40

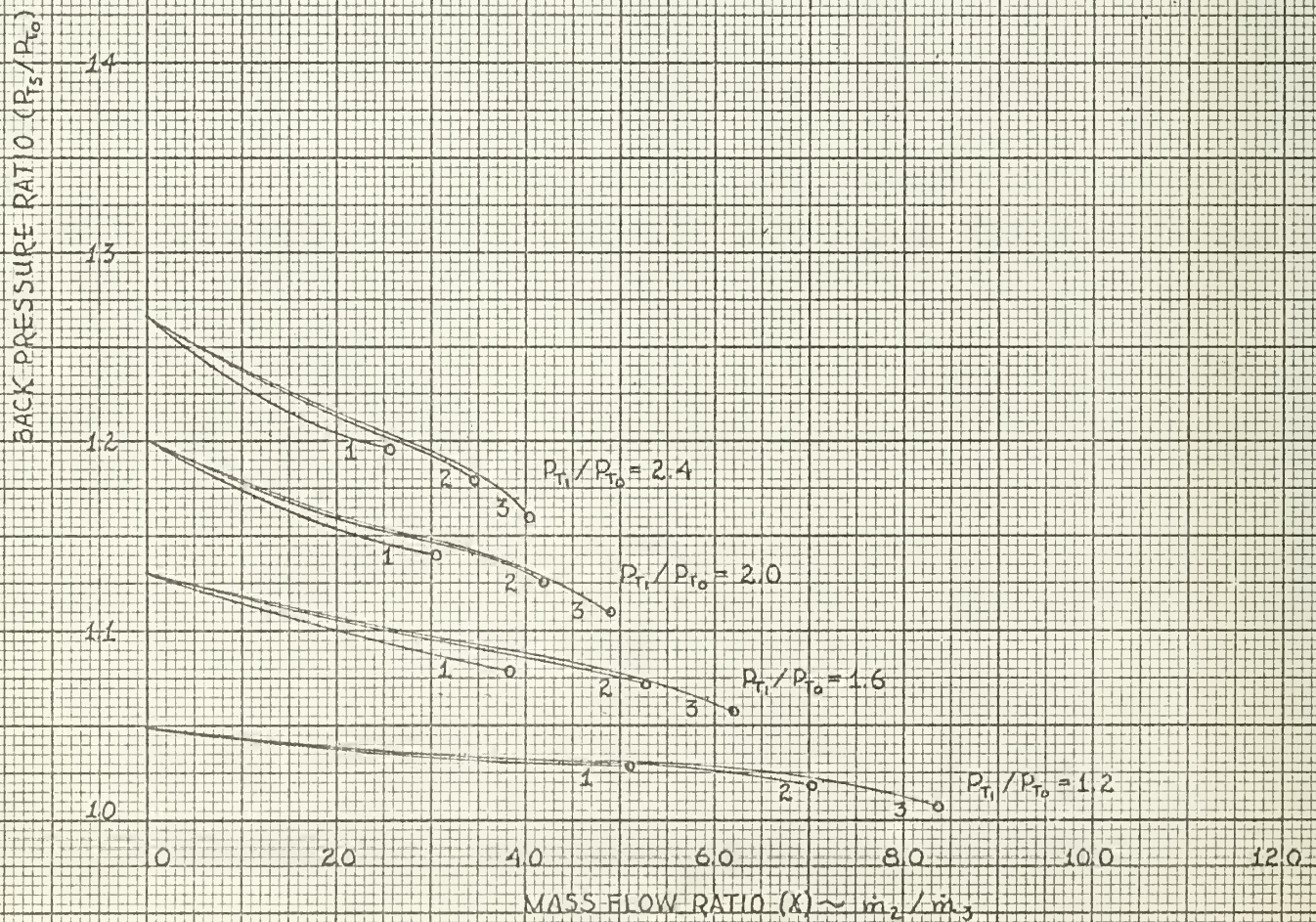
60

80

100

120

MASS FLOW RATIO ( $X$ )  $\sim m_2/m_3$





CURVE 1 DENOTES  $T_1/T_0 = 1.0$   
CURVE 2 DENOTES  $T_1/T_0 = 2.0$   
CURVE 3 DENOTES  $T_1/T_0 = 3.0$

BACK PRESSURE RATIO ( $P_r/P_o$ )





RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS		FIGURE 26
	GENERAL DESIGN CONCEPT FOR A CONSTANT AREA HEATED JETPUMP		SCALE 16:1
	JAN 1966		

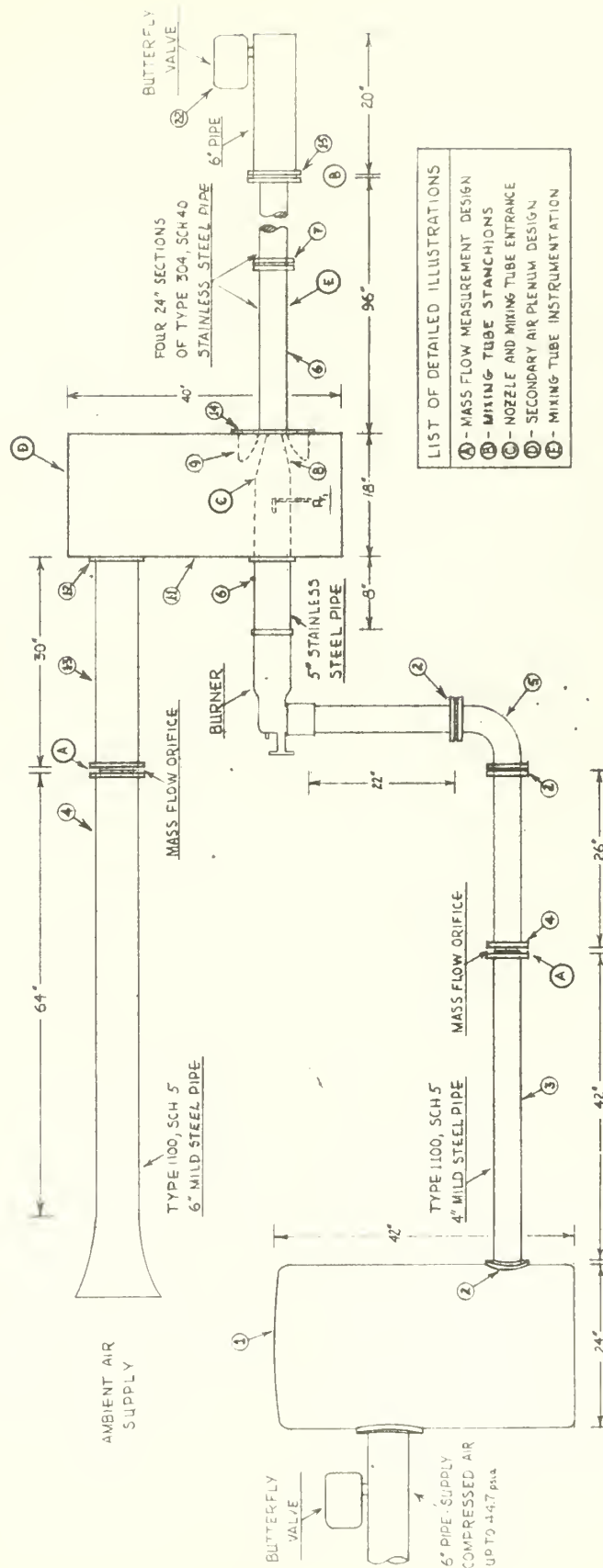




FIGURE 26	USNPGS, AERO. DEPT.
TWO INCH NOZZLE AND MIXING TUBE ENTRANCE	SCALE 2:1

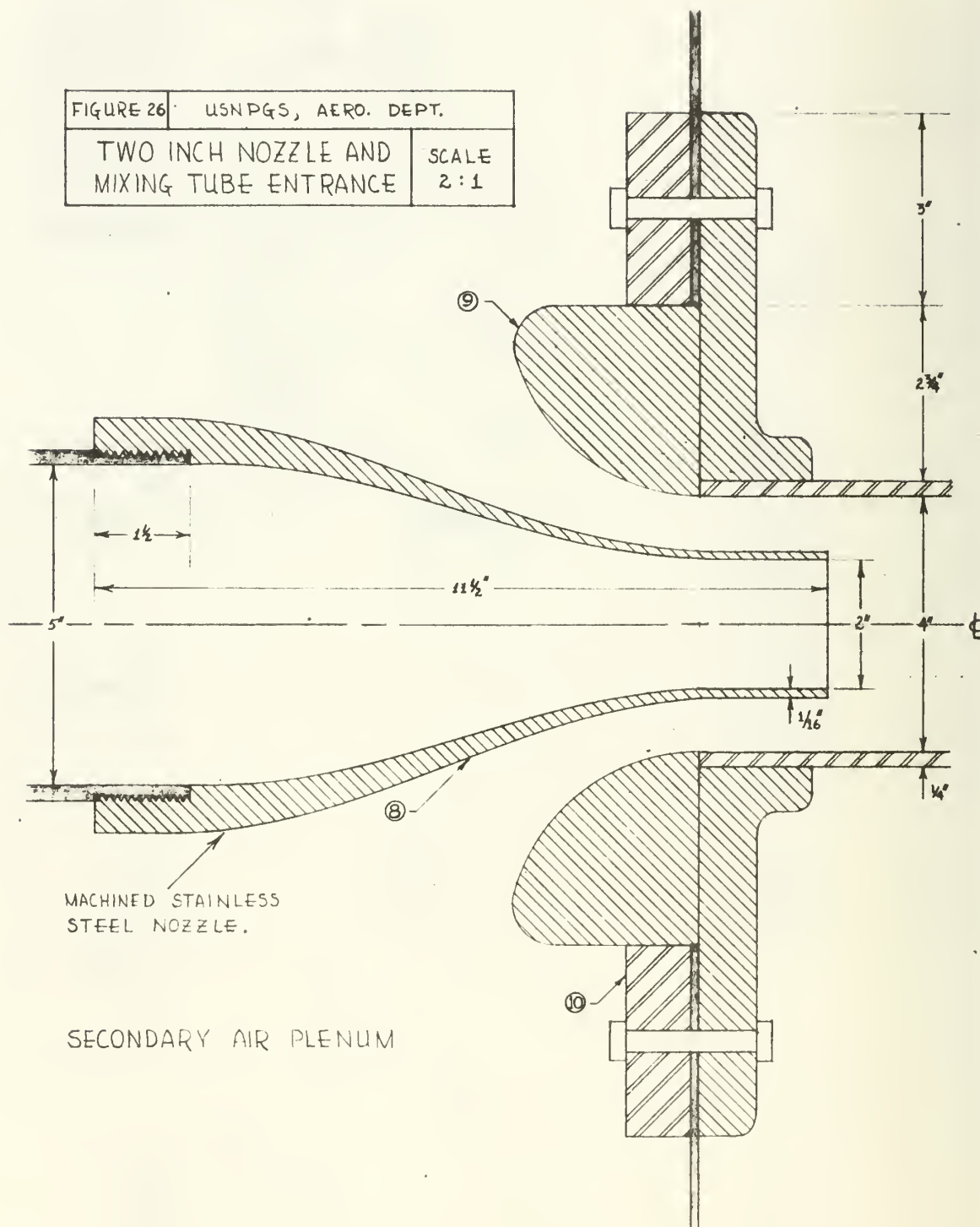


FIGURE 27	USNP&S, AERO. DEPT.
2.6 INCH NOZZLE AND MIXING TUBE ENTRANCE	SCALE 2:1

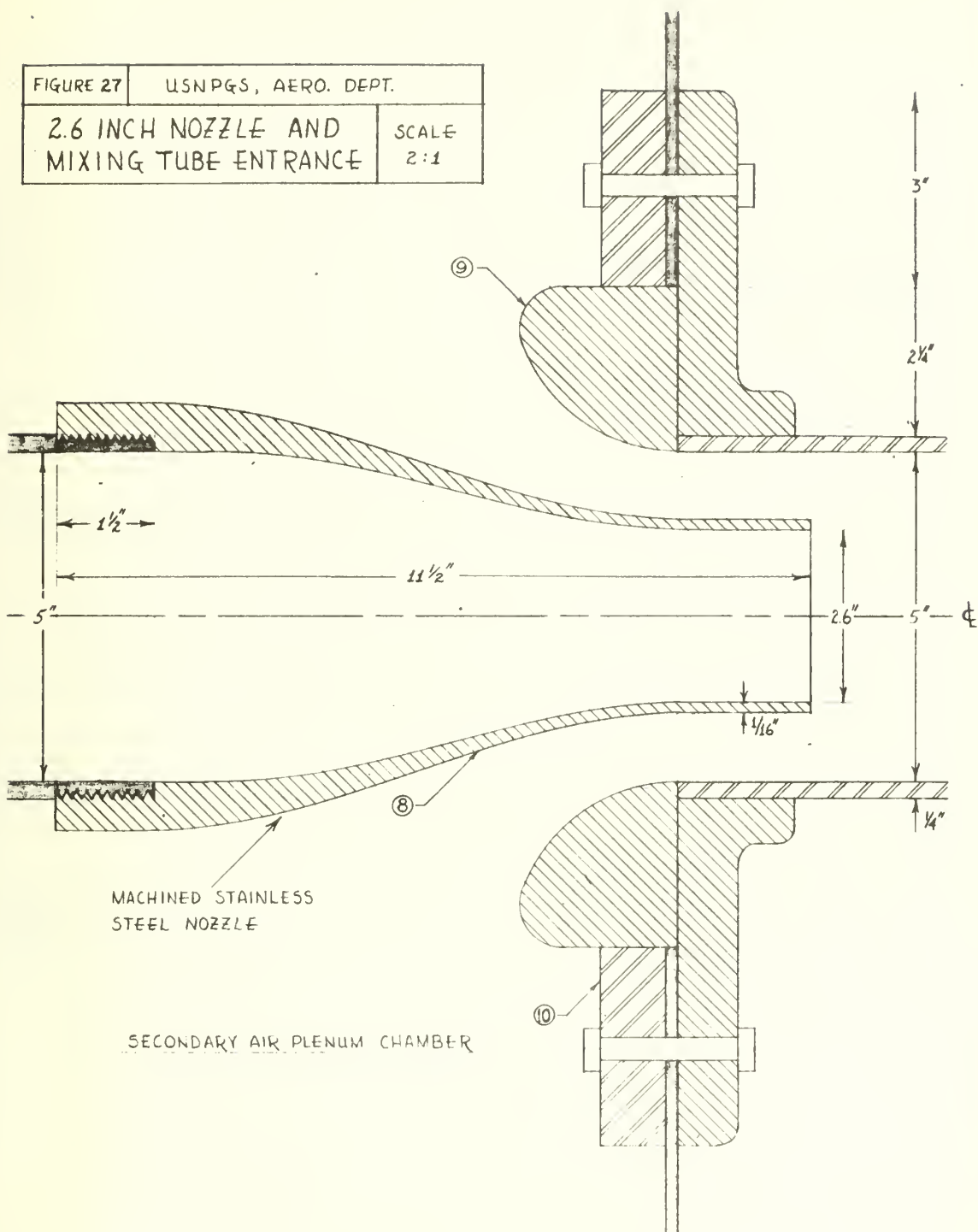
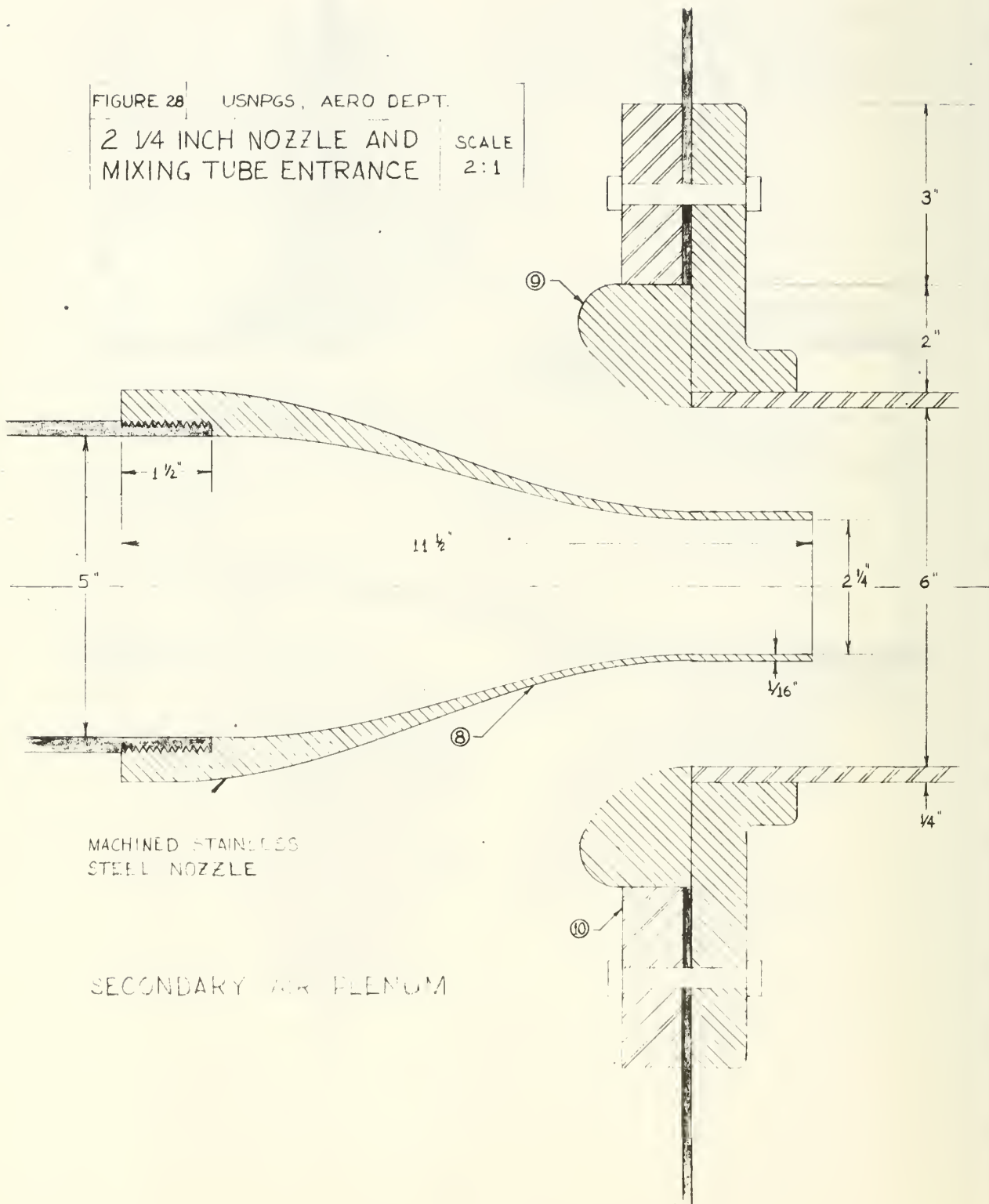


FIGURE 28 USNPGS, AERO DEPT.

2 1/4 INCH NOZZLE AND  
MIXING TUBE ENTRANCE

SCALE  
2:1





RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 30
JAN 1966	MIXING TUBE STANCHIONS AND SUPPORT DESIGN	SCALE- 8:1

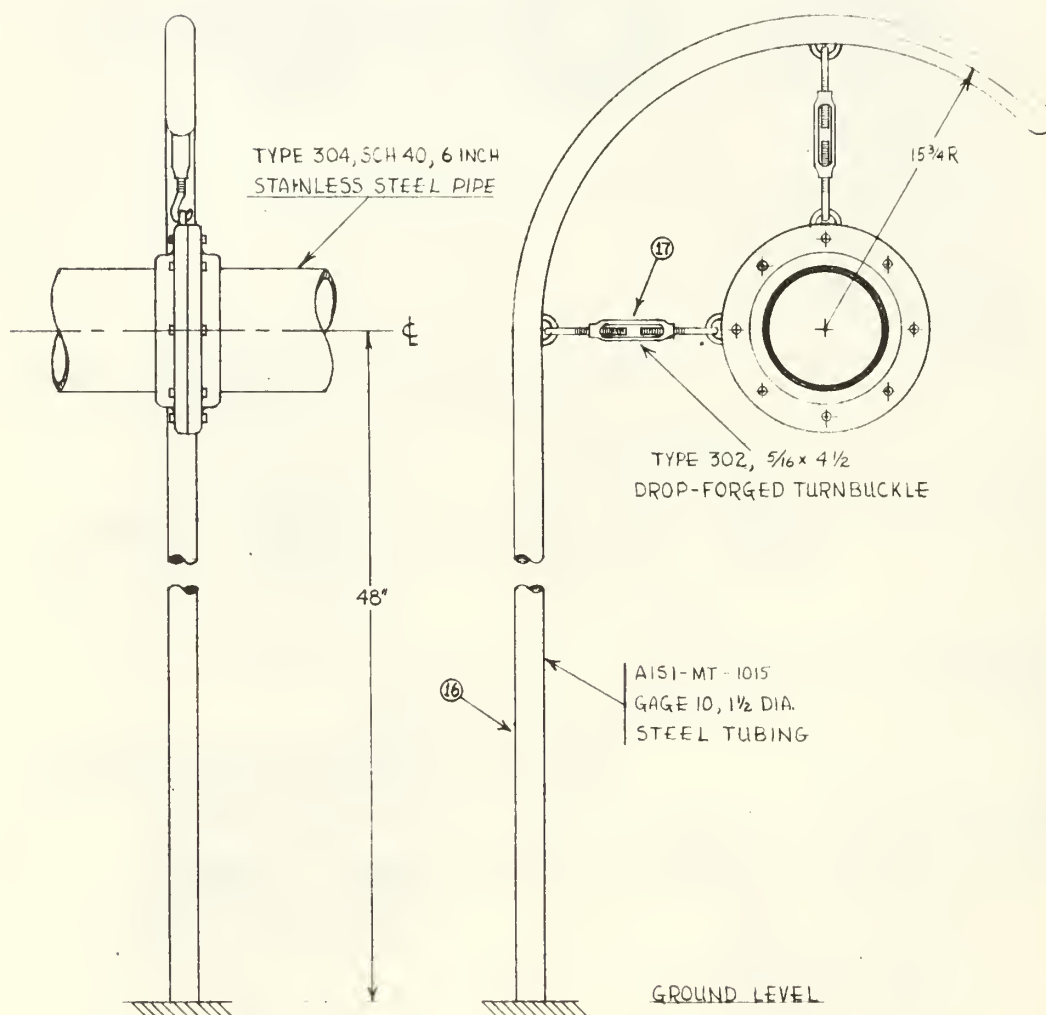
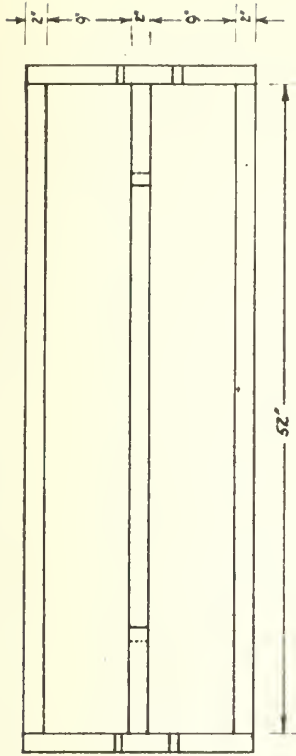
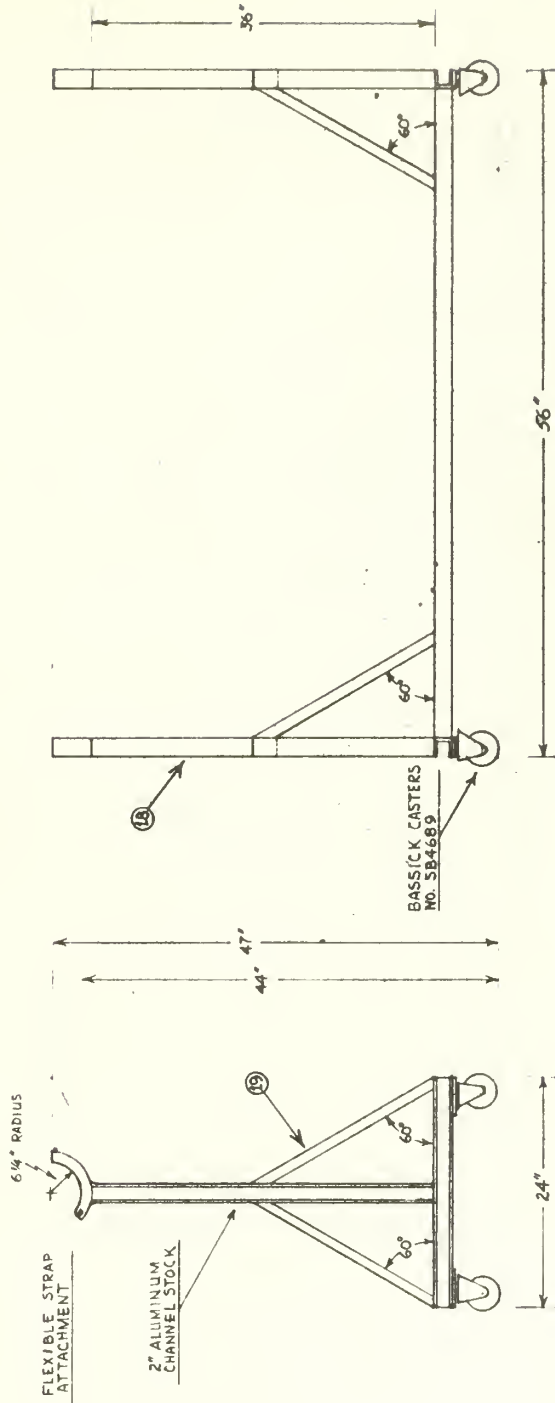




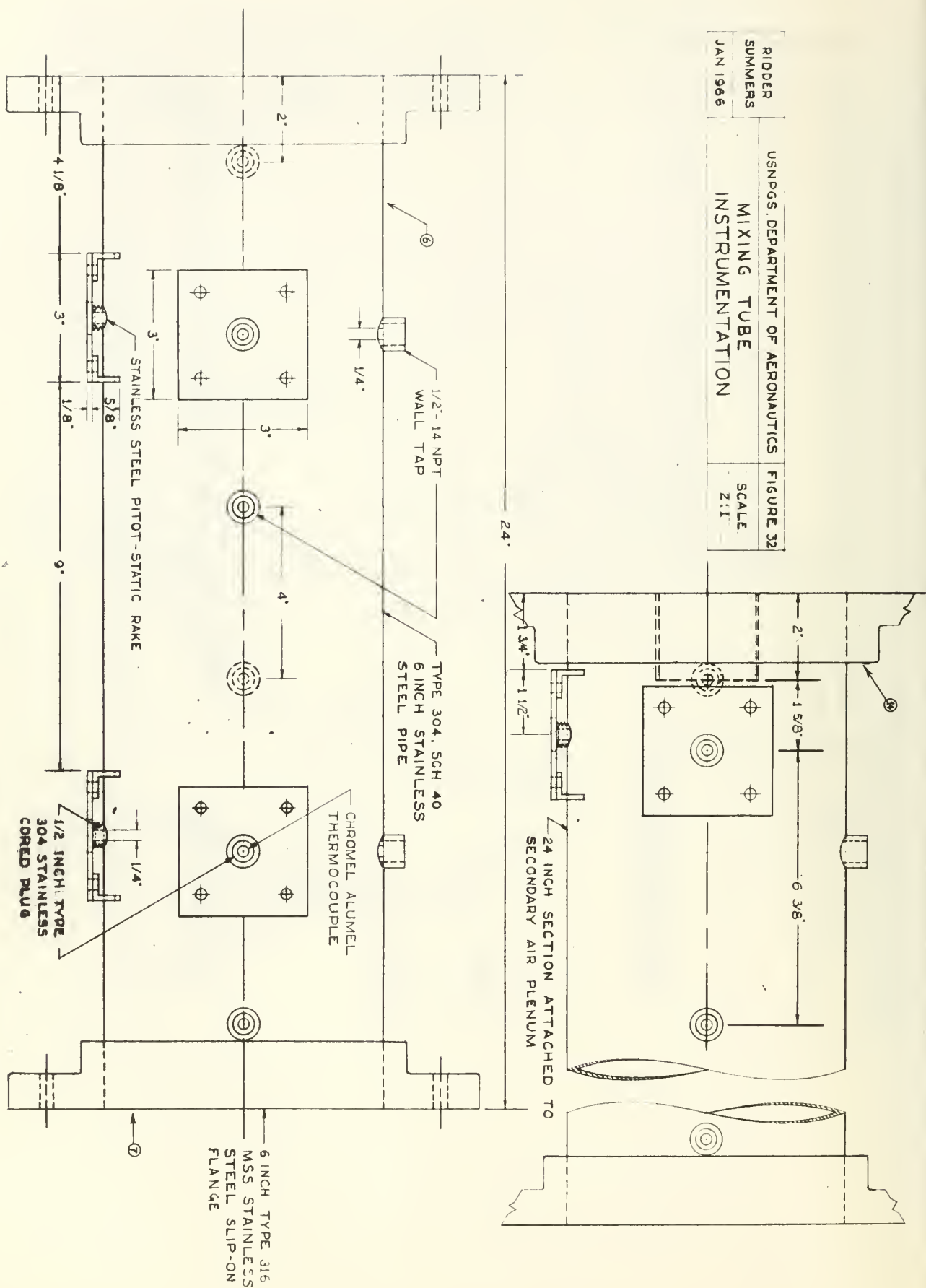
FIGURE 31	USNPGS, AERO DEPT.
MIXING TUBE HANDLING CART	SCALE 16:1



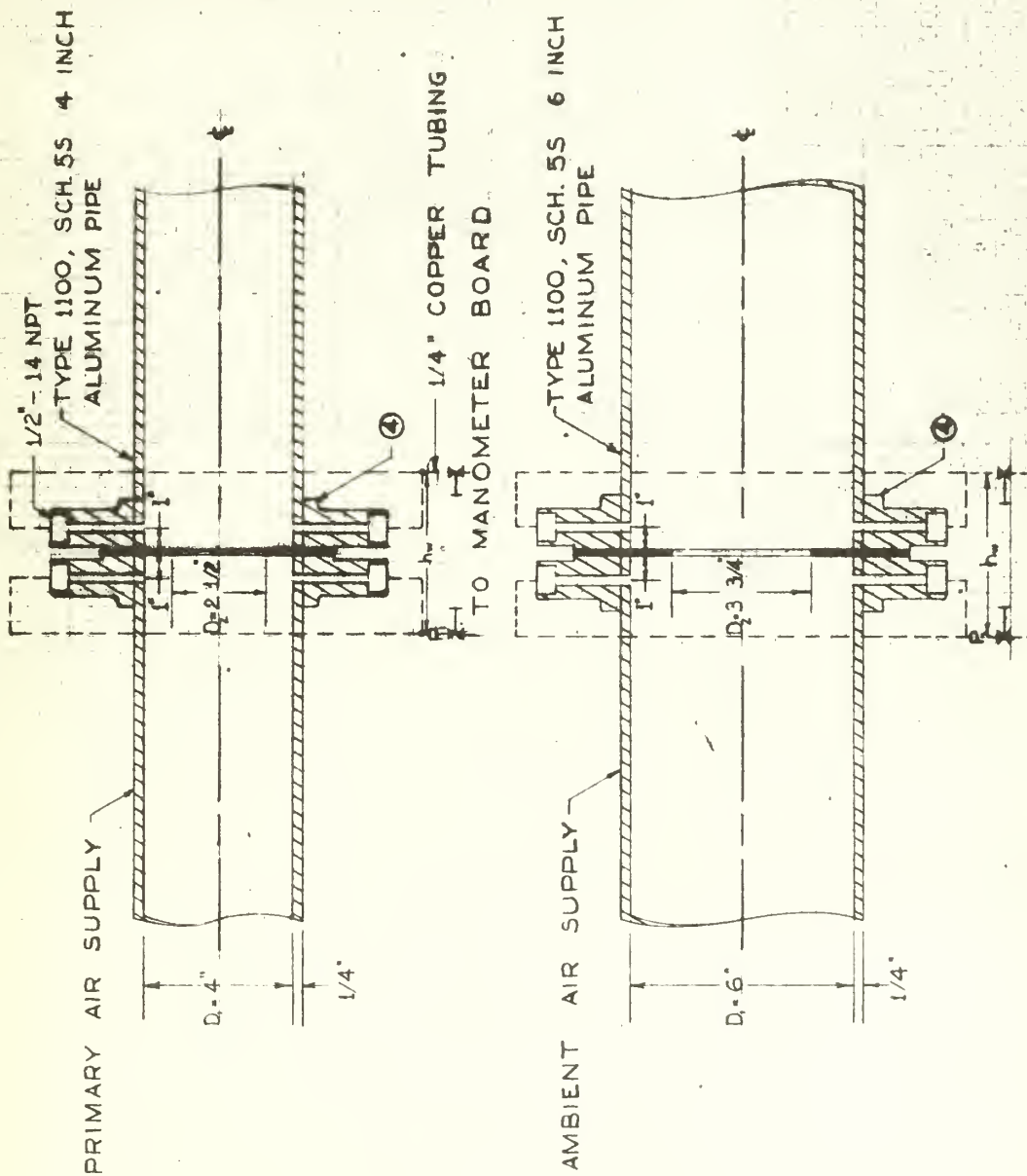
ALUMINUM CHANNEL AND BAR CONSTRUCTION  
 MAXIMUM DESIGN LOAD 1000 lbs.  
 MAXIMUM LOAD ANTICIPATED 350 lbs.



RIDDER SUMMERS JAN 1966	USNPGS. DEPARTMENT OF AERONAUTICS MIXING TUBE INSTRUMENTATION	FIGURE 32 SCALE 2:1
-------------------------------	---	---------------------------



RIDDER SUMMERS	USNPGS, DEPARTMENT OF AERONAUTICS	FIGURE 38
JAN 1966	MASS FLOW MEASUREMENT DESIGN	SCALE 4:1





## BIBLIOGRAPHY

1. Belter, R. H., Theoretical Performance of the Heated Jet Pump. M.S. Thesis, U. S. Naval Postgraduate School., 1963.
2. Gawain, T. H., Development of Non-dimensional Performance Parameters for Heated Jet Pumps. Unpublished Notes, U. S. Naval Postgraduate School.
3. Stearns, R. F., et al. Flow Measurement With Orifice Meters. D. Van Nostrand Company, 1951.
4. Helmbold, H. B., Contributions to Jet Pump Theory. University of Wichita Engineering Report No. 294, 1957.
5. Shapiro, A. H., The Dynamics and Thermodynamics of Compressible Fluid Flow. Vol. I, The Ronald Press Company, 1953.
6. Vavra, M. H., Aero-Thermodynamics and Flow in Turbo-machines. John Wiley and Sons, 1960.
7. Vavra, M. H., Determination of Flow Rates of Allis-Chalmers Axial Flow Compressor VA-312 of Propulsion Laboratories by means of Square-Edged Orifices. U. S. Naval Postgraduate School Tech Note No. 63T-2, 1963.
8. Wagner, F. G., Design and Engineering Features for Flap Suction and Combined Blowing and Suction, Boundary Layer and Flow Control, Vol. I, edited by G. V. Lachman, Pergamon Press, 1961.

## APPENDIX A

### Detailed Development

#### 1. Preliminary relationships

The preliminary relationships are similar to those of Reference

1. All velocities are arbitrarily expressed in terms of the dimensionless velocity ratio,  $M^*$ . This ratio is obtained by dividing the flow velocity,  $V$ , by the reference velocity,  $a^*$ , which is the speed of sound at sonic velocity. The Mach number,  $M$ , is equal to  $M^*$  at sonic velocity and in all cases  $aM = a^*M^*$ .  $M^*$  is also proportional to a finite velocity at all absolute temperatures.

The following equations are used throughout the development:

$$a = \sqrt{\gamma g R T}$$

$$a^* = \sqrt{\frac{2\gamma}{\gamma+1} g R T_t} = \sqrt{\gamma g R T^*}$$

$$\frac{T}{T_t} = 1 - \frac{\gamma-1}{\gamma+1} M^{*2}$$

$$\frac{p}{p_t} = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p}{p_t} = \left(\frac{T}{T_t}\right)^{\frac{1}{\gamma-1}}$$

$$\dot{m}_2 = \rho_2 A_2 V_2$$

$$\dot{m}_3 = \rho_3 A_3 V_3$$

$$C_P = \frac{R}{J} \frac{\gamma}{\gamma-1}$$

$$\gamma = \frac{C_P}{C_V}$$

$$p = \rho g R T$$

## 2. Performance coefficients

### a. Momentum coefficient

The momentum coefficient is defined as the dimensionless momentum:

$$C_M = \frac{\rho_5 A_5 V_5^2}{\rho_o^* A_6 a_o^{*2}} \quad (A-1)$$

Substituting  $a_5^{*2} M_5^{*2}$  for  $V_5^2$

$$C_M = \frac{\rho_5 A_5 a_5^{*2} M_5^{*2}}{\rho_o^* A_6 a_o^{*2}} \quad (A-2)$$

Now  $\rho_5 a_5^{*2} M_5^{*2}$  can be written

$$\rho_5 a_5^{*2} M_5^{*2} = \frac{\rho_5}{\rho_5^*} \rho_5^* a_5^{*2} M_5^{*2} \quad (A-3)$$

where

$$\frac{\rho_5}{\rho_5^*} = \left( \frac{T_5}{T_5^*} \right)^{\frac{1}{\gamma-1}}$$

which can be written

$$\frac{\rho_5}{\rho_5^*} = \left( \frac{T_5/T_{t5}}{T_5^*/T_{t5}} \right)^{\frac{1}{\gamma-1}} \quad (A-4)$$



Substituting for  $T_5/T_{t5}$  and  $T_5^*/T_{t5}$  into (A-4)

$$\frac{\rho_5}{\rho_5^*} = \left( \frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}} \right)^{\frac{1}{\gamma-1}}$$

$\rho_5^* a_5^{*2}$  can be written in terms of the equation of state and the speed of sound at sonic velocity.

$$\rho_5^* a_5^{*2} = \left( \frac{p_5^*}{\gamma R T_5^*} \right) (\gamma g R T_5^*)$$

which can be written

$$\rho_5^* a_5^{*2} = \gamma \left( \frac{p_5^*}{P_{t5}} \right) P_{t5}$$

Since  $p_5^*$  corresponds to static pressure at  $M_5 = M_5^* = 1$ , then,

$$\rho_5^* a_5^{*2} = \gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} P_{t5}$$

Substituting for  $\rho_5/\rho_5^*$  and  $\rho_5^* a_5^{*2}$  into (A-3)

$$\rho_5 a_5^{*2} M_5^{*2} = \left( \frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}} \right)^{\frac{1}{\gamma-1}} \gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} P_{t5} M_5^{*2}$$

Similar to  $\rho_5^* a_5^{*2}$

$$\rho_0^* a_0^{*2} = \gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} P_{t0}$$

Dividing  $\rho_5 a_5^{*2} M_5^{*2}$  by  $\rho_0^* a_0^{*2}$

$$\frac{\rho_5 a_5^{*2} M_5^{*2}}{\rho_0^* a_0^{*2}} = \left( \frac{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}}{\frac{2}{\gamma+1}} \right)^{\frac{1}{\gamma-1}} \frac{P_{t5}}{P_{t0}} M_5^{*2}$$

From continuity and remembering  $M_6^* = 1$

$$\rho_5 A_5 V_5 = \rho_6 A_6 V_6$$

$$\frac{A_5}{A_6} = \frac{\rho_6 V_6}{\rho_5 V_5}$$

Substituting for  $V_5$  and  $V_6$

$$\frac{A_5}{A_6} = \frac{P_6 M_6^* a_6^*}{P_5 M_5^* a_5^*} \quad (A-5)$$

Since  $T_{t6} = T_{t5}$ , therefore  $a_6^* = a_5^*$ , hence

$$\frac{A_5}{A_6} = \frac{P_6}{P_5 M_5^*}$$

Substituting for  $(P_6/P_{t6})/(P_5/P_{t5})$  in (A-5) and noting  $P_{t6} = P_{t5}$

$$\frac{A_5}{A_6} = \left( \frac{1}{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}} \right)^{\frac{\gamma+1}{2}} \frac{1}{M_5^*}$$

Substituting into Equation (A-2) for

$$\frac{P_5 a_5^* M_5^*}{P_0^* a_0^*}$$

and  $A_5/A_6$ , the result is

$$C_M = \frac{P_{t5}}{P_{t0}} M_5^*$$

and substituting for  $M_5^*$

$$C_M = \frac{P_{t5}}{P_{t0}} \left\{ \frac{\gamma+1}{\gamma-1} \left[ 1 - \left( \frac{P_{t0}}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

It is seen that  $C_M$  is a function only of the required discharge total pressure ratio. In particular,  $C_M$  is independent of the amount of heat supplied. By specifying the desired output pressure ratio,  $P_{t5}/P_{t0}$ , the required  $C_M$  is determined.

#### b. Power-momentum coefficient

The ratio of the power coefficient to the momentum coefficient defines the power-momentum coefficient.

$$C_{PM} = \frac{C_P}{C_M} = \frac{P_3 A_3 V_3 a_0^* c_p \Delta T_c}{P_5 A_5 V_5^2 \frac{a_0^{*2}}{2gJ}}$$

$$C_{PM} = \frac{\dot{m}_3}{\dot{m}_2 + \dot{m}_3} \frac{\Delta H_c 2gJ}{V_5 a_0^*}$$

$$C_{PM} = \frac{\Delta H_c \ 2gJ}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) V_s a_o^*} \quad (A-6)$$

$$\Delta H_c = c_p (T_{t1c} - T_{t0})$$

$$\Delta H_c = c_p T_{t0} \left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

Substituting for  $\Delta H_c$  and  $V_s$  in (A-6), then  $C_{PM}$  becomes

$$C_{PM} = \frac{c_p T_{t0} \left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} 2gJ}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) a_s^* M_s^* a_o^*}$$

also

$$\frac{2gJc_p T_{t0}}{a_s^* a_o^*} = \frac{2gJR\gamma T_{t0}}{\left(\frac{2\gamma}{\gamma+1} gRT_{t5}\right)^{\frac{1}{2}} \left(\frac{2\gamma}{\gamma+1} gRT_{t0}\right)^{\frac{1}{2}} J(\gamma-1)} \quad (A-7)$$

Therefore

$$C_{PM} = \frac{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \frac{\gamma+1}{\gamma-1}}{\left(\frac{\dot{m}_2}{\dot{m}_3} + 1\right) \sqrt{\frac{T_{t5}}{T_{t0}}} M_s^*} \quad (A-8)$$

rearranging and substituting in (A-8) for  $T_{t5}/T_{t0}$  from the energy equation (A-31)

$$\frac{C_{PM}}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} = \frac{\frac{\gamma+1}{\gamma-1} (M_s^*)^2}{\left\{ (\gamma+1) \left( \gamma + \frac{1}{T_{t0}} \right) \right\}} \quad (A-9)$$

#### c. Heat-momentum coefficient

The ratio of the heat coefficient to the momentum coefficient



defines the heat-momentum coefficient.

$$C_{QM} = \frac{C_Q}{C_M} = \frac{\rho_3 A_3 V_3 a_o^*}{\rho_5 A_5 V_5^2} \frac{c_p \Delta T_B}{\frac{a_o^{*2}}{2gJ}}$$

$$C_{QM} = \frac{\dot{m}_3}{\dot{m}_2 + \dot{m}_3} \frac{\Delta q 2gJ}{V_5 a_o^*}$$

$$C_{QM} = \frac{\Delta q}{(\chi + 1)} \frac{2gJ}{a_5^* M_5^* a_o^*}$$

$$\Delta q = c_p (T_{t1} - T_{t1c})$$

$$\Delta q = c_p T_{t0} \left( \frac{T_{t1}}{T_{t0}} - \frac{T_{t1c}}{T_{t0}} \right)$$

Since there is an isentropic relationship between state O and 1c, then  $\Delta q$  becomes

$$\Delta q = c_p T_{t0} \left\{ \frac{T_{t1}}{T_{t0}} - \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right\}$$

Following a similar substitution for  $\frac{2gJ c_p T_{t0}}{a_5^* a_o^*}$  as in

(A-7)  $C_{QM}$  becomes:

$$C_{QM} = \frac{\left\{ \frac{T_{t1}}{T_{t0}} - \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right\} \frac{\gamma+1}{\gamma-1}}{(\chi+1) \sqrt{\frac{T_{t5}}{T_{t0}}} M_5^*} \quad (A-10)$$

Rearranging and substituting for  $T_{t5}/T_{t0}$  from the energy equation (A-31)

$$\frac{C_{QM}}{\left\{ \frac{T_{t1}}{T_{t0}} - \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right\}} = \frac{\frac{\gamma+1}{\gamma-1} (M_5^*)^{-1}}{\left\{ (\chi+1) \left( \chi + \frac{T_{t1}}{T_{t0}} \right) \right\}^{1/2}} \quad (A-11)$$

It is noted that the following equation can be written involving  $C_{QM}$  and  $C_{PM}$  from (A-9) and (A-11).

$$\frac{C_{PM}}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} = \frac{C_{QM}}{\left\{ \frac{T_{t1}}{T_{t0}} - \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right\}} = \frac{\frac{\gamma+1}{\gamma-1} (M_5^*)^{-1}}{\left\{ (\kappa+1) \left( \kappa + \frac{T_{t1}}{T_{t0}} \right) \right\}^{1/2}} \quad (A-12)$$

Consider the case where  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ , and  $P_{t5}/P_{t0}$  are fixed. This equation shows that under these conditions  $C_{PM}$  and  $C_{QM}$  will be a minimum when the mass flow ratio,  $\dot{m}_1/\dot{m}_3 = \kappa$ , is a maximum. Also, it is noted that this equation is invariant for both the constant pressure and constant area cases, although the maximum value of  $\kappa$  may differ in the two cases.

#### d. Efficiency

Another performance parameter of some interest is the mechanical efficiency which is, of course, based on energy consideration, namely,

$$\eta_M = \frac{\text{Final available energy (Isentropic)}}{\text{Initial available energy (Isentropic)}}$$

$$\eta_M = \frac{\dot{m}_5 (H_5 - h_5)}{\dot{m}_3 \Delta H_c} = \frac{\rho_5 A_5 V_5}{\rho_3 A_3 V_3} \cdot \frac{V_5^2 / 2gJ}{c_p \Delta T_c}$$

$$\eta_M = \frac{1}{\frac{\rho_3 A_3 V_3}{\rho_5 A_5 V_5}} \cdot \frac{\frac{V_5}{a_o^*} \frac{c_p \Delta T_c}{V_5} a_o^*}{\frac{c_p \Delta T_c}{a_o^* / 2gJ} a_o^*} = \frac{V_5}{C_{PM} a_o^*}$$

$$\eta_M = \frac{a_5^* V_5}{C_{PM} a_o^* a_5^*} = \frac{1}{C_{PM}} \sqrt{\frac{T_{t5}}{T_{t0}}} M_5^*$$

Substituting for  $C_{PM}$  from (A-8)

$$\eta_M = \frac{(\chi + 1) \sqrt{\frac{T_{t5}}{T_{t0}}} M_5^*}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \frac{\gamma+1}{\gamma-1} \sqrt{\frac{T_{t5}}{T_{t0}}} M_5^*}$$

Substituting for  $T_{t5}/T_{t0}$  from the energy equation (A-31).

$$\eta_M = \frac{\left( \frac{\gamma-1}{\gamma+1} \right) M_5^{*2}}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} \left( \chi + \frac{T_{t1}}{T_{t0}} \right) \quad (A-13)$$

Since,  $p_5 = P_{t0}$ , then

$$\frac{\gamma-1}{\gamma+1} M_5^{*2} = 1 - \left( \frac{P_{t0}}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}}$$

Substituting for  $\frac{\gamma-1}{\gamma+1} M_5^*$  into (A-13)

Therefore:

$$\eta_M = \frac{\left\{ 1 - \left( \frac{P_{t0}}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right\}}{\left\{ \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}} \left( \chi + \frac{T_{t1}}{T_{t0}} \right) \quad (A-14)$$

This equation shows that  $\eta_M$ , like the other performance parameters, will be optimum when  $\chi$  is a maximum for specified conditions of pressure and temperature.

### 3. Development of constant pressure jet pump theory

An explicit solution to the equations of continuity, energy, and momentum is possible when the following parameters are considered fixed:

$P_{t5}/P_{t0}$	the ratio of the demanded discharge total pressure to the ambient pressure.
$P_{t1}/P_{t0}$	the ratio of the primary (jet) total pressure to the ambient pressure.
$T_{t1}/T_{t0}$	the ratio of the primary (jet) total temperature to the ambient temperature.
$M_2^*$	the dimensionless secondary velocity.



The usual isentropic relationships are used to solve for conditions in the primary (jet) system and in the secondary jet system up to the mixing tube entrance.

The critical value of  $M_2^*$  for choking of the primary nozzle can be determined from the isentropic relationships

$$\frac{p_3}{p_{t1}} = \frac{p_3}{p_{t3}} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M_3^{*2} \right]^{\frac{\gamma}{\gamma-1}} \quad (\text{A-15})$$

$$\frac{p_2}{p_{t0}} = \frac{p_2}{p_{t2}} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M_2^{*2} \right]^{\frac{\gamma}{\gamma-1}} \quad (\text{A-16})$$

For sub-critical flow,  $M_3^* < 1.0$  and  $M_2^* < M_{2cr}^*$ , the primary flow is unchoked and  $p_2 = p_3$ .  $M_{2cr}^*$  is the critical secondary velocity ratio at which  $M_3^*$  becomes sonic for supply pressure ratios less 1.89. Multiplying  $p_2/p_{t0}$  by  $p_{t1}/p_3$  from (A-15) and (A-16) results in

$$\frac{p_{t1}}{p_{t0}} = \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_{2cr}^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}} \right\}^{\frac{\gamma}{\gamma-1}} \quad (\text{A-17})$$

Solving for  $M_3^*$  from (A-17)

$$M_3^* = \left\{ \frac{\gamma+1}{\gamma-1} \left[ 1 - \frac{\left[ 1 - \frac{\gamma-1}{\gamma+1} M_{2cr}^{*2} \right]^{\frac{\gamma}{\gamma-1}}}{\left( \frac{p_{t1}}{p_{t0}} \right)^{\frac{\gamma}{\gamma-1}}} \right] \right\}^{\frac{1}{2}} \quad (\text{A-18})$$

Also

$$\frac{p_3}{p_{t0}} = \frac{p_2}{p_{t0}} = \frac{p_2}{p_{t2}} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M_2^{*2} \right]^{\frac{\gamma}{\gamma-1}}$$

At critical conditions,  $M_3^* = 1$ ,  $M_2^* = M_{2cr}^*$ , and  $p_2 = p_3$ .

It is possible to determine the critical  $M_2^*$  as a function of  $p_{t1}/p_{t0}$  from (A-17)

$$\frac{2}{\gamma+1} \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma-1}{\gamma+1} M_{2cr}^{*2}$$

$$M_{2cr}^* = \left\{ \frac{\gamma+1}{\gamma-1} \left[ 1 - \frac{2}{\gamma+1} \left( \frac{P_{t1}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}} \quad (A-19)$$

For super-critical flow,  $M_3^* = 1$ ,  $M_2^* > M_{2cr}^*$ , and  $p_2 \neq p_3$ . For supply pressure ratios,  $P_{t1}/P_{t0}$ , greater than 1.89,  $M_3^* = 1.0$  for all conditions. This renders the concept of  $M_{2cr}^*$  meaningless and  $p_2$  is not equal to  $p_3$ , specifically  $p_2 < p_3$ .

$$\frac{p_3}{P_{t0}} = \left( \frac{p_2}{P_{t1}} \right) \left( \frac{P_{t1}}{P_{t0}} \right)$$

$$\frac{p_3}{P_{t1}} = \frac{p_2}{P_{t3}} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M_3^{*2} \right]$$

Since  $M_3^* = 1.0$ ,

$$\frac{p_3}{P_{t0}} = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{P_{t1}}{P_{t0}} \right) \quad (A-20)$$

and

$$\frac{p_2}{P_{t0}} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M_2^{*2} \right]^{\frac{\gamma}{\gamma-1}}$$

The basic condition for constant pressure mixing is:

$$p_2 = p_4 \quad (A-21)$$

Therefore  $\frac{p_2}{P_{t0}} = \left( \frac{p_4}{P_{t4}} \right) \left( \frac{P_{t5}}{P_{t0}} \right)$  since  $P_{t4} = P_{t5}$

Substituting the isentropic relationships for  $p_2/P_{t0}$  and  $P_{t5}/P_{t0}$

(  $P_{t5} = P_{t0}$  ) and raising to the  $\frac{\gamma-1}{\gamma}$  power.

$$\left[ 1 - \frac{\gamma-1}{\gamma+1} M_2^{*2} \right] = \left( \frac{P_{t5}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}} \left[ 1 - \frac{\gamma-1}{\gamma+1} M_4^{*2} \right]$$

Hence  $M_4^*$  can be determined from

$$M_4^* = \left\{ \frac{\gamma+1}{\gamma-1} \left[ 1 - \frac{\left[ 1 - \frac{\gamma-1}{\gamma+1} M_2^{*2} \right]}{\left( \frac{P_{t5}}{P_{t0}} \right)^{\frac{\gamma-1}{\gamma}}} \right] \right\}^{\frac{1}{2}} \quad (\text{A-22})$$

Analyzing the Equations (A-18) and (A-22), it is seen that  $M_3^*$  and  $M_4^*$  are fixed when  $P_{t1}/P_{t0}$ ,  $P_{t5}/P_{t0}$  and  $M_2^*$  are fixed.  $T_{t1}/T_{t0}$  does not appear in either equation.

The three basic equations that have to be satisfied for constant pressure mixing ( $P_2 = P_4$ ) are continuity, energy and momentum.

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \quad (\text{A-23})$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \quad (\text{A-24})$$

Momentum:

$$(\dot{m}_3 + \dot{m}_2) V_4 - (\dot{m}_3 V_3 + \dot{m}_2 V_2) = (p_3 - p_2) A_3 \quad (\text{A-25})$$

It is noted that the resultant pressure forces in (A-25) are zero in the constant pressure case when  $M_3^* \leq 1$  and  $M_2^* \leq M_{2cr}^*$  since  $p_2 = p_3$ . When  $M_3^* = 1$  and  $M_2^* > M_{2cr}^*$  then  $p_2 < p_3$ . The continuity equation (A-23) can be rearranged as follows:

$$\frac{\dot{m}_2}{\dot{m}_3} + 1 = \frac{\dot{m}_4}{\dot{m}_3}$$

Let  $\chi = \dot{m}_2 / \dot{m}_3$

So,

$$\begin{aligned} \chi + 1 &= \frac{P_4}{P_3} \frac{A_4}{A_3} \frac{M_4^*}{M_3^*} \frac{a_4^*}{a_3^*} \\ \frac{A_4}{A_3} &= (\chi + 1) \frac{P_3}{P_4} \frac{M_3^*}{M_4^*} \frac{a_3^*}{a_4^*} \end{aligned} \quad (\text{A-26})$$



and from the isentropic relationship:

$$\frac{p}{p_t} = \left[ 1 - \frac{\gamma-1}{\gamma+1} M^{*2} \right]^{\frac{1}{\gamma-1}}$$

$$\frac{p_3}{p_4} = \frac{p_{t3}}{p_{t4}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}} = \frac{\frac{p_{t3}}{T_{t3}}}{\frac{p_{t5}}{T_{t5}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}}$$

$$\frac{\frac{p_{t3}}{T_{t3}}}{\frac{p_{t5}}{T_{t5}}} = \frac{p_{t3}}{p_{t5}} \frac{T_{t5}}{T_{t3}} = \frac{\frac{p_{t1}}{p_{t0}} \frac{T_{t5}}{T_{t0}}}{\frac{p_{t5}}{p_{t0}} \frac{T_{t1}}{T_{t0}}}$$

Hence,

$$\frac{p_3}{p_4} = \frac{T_{t5}}{T_{t0}} \frac{p_{t1}}{p_{t0}} \frac{1}{\frac{T_{t1}}{T_{t0}} \frac{p_{t5}}{p_{t0}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}}$$

$$\frac{a_3^*}{a_4^*} = \left( \frac{T_{t3}}{T_{t4}} \right)^{\frac{1}{2}} = \left( \frac{T_{t5}}{T_{t1}} \frac{T_{t1}}{T_{t0}} \frac{T_{t0}}{T_{t4}} \right)^{\frac{1}{2}} = \left( \frac{T_{t1}}{T_{t0}} \frac{T_{t0}}{T_{t4}} \right)^{\frac{1}{2}}$$

Substituting for  $p_3/p_4$  and  $a_3^*/a_4^*$  into Equation (A-26).

$$\frac{A_4}{A_3} = (\gamma+1) \left( \frac{T_{t4}}{T_{t0}} \right)^{\frac{1}{2}} \frac{\frac{p_{t1}}{p_{t0}}}{\left( \frac{p_{t5}}{p_{t0}} \right) \left( \frac{T_{t1}}{T_{t0}} \right)^{\frac{1}{2}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*}$$

It will be shown from the energy equation (A-31) that;

$$\frac{T_{t5}}{T_{t0}} = \frac{T_{t4}}{T_{t0}} = \frac{\left( \frac{T_{t1}}{T_{t0}} + \gamma \right)}{1 + \gamma}$$

Substituting for  $T_{t4}/T_{t0}$ , then  $A_4/A_3$  becomes;

$$\frac{A_4}{A_3} = \left[ (\gamma+1) \left( \frac{T_{t1}}{T_{t0}} + \gamma \right) \right]^{\frac{1}{2}} \frac{\frac{p_{t1}}{p_{t0}}}{\frac{p_{t5}}{p_{t0}} \left( \frac{T_{t1}}{T_{t0}} \right)^{\frac{1}{2}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_4^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_4^*} \quad (\text{A-27})$$

This is the dimensionless form of the continuity equation.

Similarly replacing  $M_4^*$  by  $M_5^*$  and noting that  $T_{t5} = T_{t4}$  and  $P_{t5} = P_{t4}$  then;

$$\frac{A_5}{A_3} = \left[ (\gamma + 1) \left( \frac{T_{t1}}{T_{t0}} + \gamma \right) \right]^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{t0}}}{\frac{P_{t5}}{P_{t0}} \left( \frac{T_{t1}}{T_{t0}} \right)^{\frac{1}{2}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_5^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_5^*} \quad (\text{A-28})$$

Similarly for  $A_6/A_3$  replacing  $M_5^*$  by  $M_6^* = 1$

$$\frac{A_6}{A_3} = \left[ (\gamma + 1) \left( \frac{T_{t1}}{T_{t0}} + \gamma \right) \right]^{\frac{1}{2}} \frac{\frac{P_{t1}}{P_{t0}}}{\frac{P_{t5}}{P_{t0}} \left( \frac{T_{t1}}{T_{t0}} \right)^{\frac{1}{2}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{\frac{2}{\gamma+1}} \right\}^{\frac{1}{\gamma-1}} M_3^* \quad (\text{A-29})$$

From the mass flow ratio  $\gamma$ , it is possible to determine  $A_2/A_3$ .

$$\frac{\dot{m}_2}{\dot{m}_3} = \gamma = \frac{\rho_2 A_2 a_2^* M_2^*}{\rho_3 A_3 a_3^* M_3^*}$$

and:

$$\frac{A_2}{A_3} = \gamma \frac{\frac{P_{t1}}{P_{t0}}}{\left( \frac{T_{t1}}{T_{t0}} \right)^{\frac{1}{2}}} \left\{ \frac{1 - \frac{\gamma-1}{\gamma+1} M_3^{*2}}{1 - \frac{\gamma-1}{\gamma+1} M_2^{*2}} \right\}^{\frac{1}{\gamma-1}} \frac{M_3^*}{M_2^*} \quad (\text{A-30})$$

Consider the energy equation, (A-24).

$$\dot{m}_2 c_p T_{t2} + \dot{m}_3 c_p T_{t3} = \dot{m}_4 c_p T_{t4}$$

$$\dot{m}_2 T_{t2} + \dot{m}_3 T_{t3} = \dot{m}_4 T_{t4}$$

$$\frac{\dot{m}_2}{\dot{m}_3} + \frac{T_{t1}}{T_{t0}} = \left( \frac{\dot{m}_2}{\dot{m}_3} + 1 \right) \frac{T_{t4}}{T_{t0}}$$

The non-dimensional form of energy equation is

$$\left( \frac{T_{t1}}{T_{t0}} + \gamma \right) = (1 + \gamma) \frac{T_{t4}}{T_{t0}} \quad (\text{A-31})$$

Now consider the momentum equation (A-25). Substituting for the velocities and rearranging

$$\dot{m}_3 a_3^* M_3^* + \dot{m}_2 a_2^* M_2^* + (p_3 - p_2) A_3 = (\dot{m}_3 + \dot{m}_2) a_4^* M_4^*$$

Dividing both sides by  $\dot{m}_3 a_o^*$ .

$$\frac{a_3^*}{a_o^*} M_3^* + \left( \frac{\dot{m}_2}{\dot{m}_3} \right) \frac{a_2^*}{a_o^*} M_2^* + \frac{(p_3 - p_2) A_3}{a_o^* \dot{m}_3} = \left( 1 + \frac{\dot{m}_2}{\dot{m}_3} \right) \frac{a_4^*}{a_o^*} M_4^* \quad (A-32)$$

Let

$$D = \frac{(p_3 - p_2) A_3}{a_o^* \dot{m}_3}$$

D can be expressed as follows

$$D = \left( 1 - \frac{p_2}{p_3} \right) \frac{p_3}{a_o^*} \frac{1}{\rho_3 v_3} = \left( 1 - \frac{p_2}{p_3} \right) \frac{p_3}{a_o^*} \frac{g R T_3}{\rho_3 a_3^* M_3^*}$$

$$D = \left( 1 - \frac{p_2}{p_3} \right) \frac{g R T_3}{a_o^*} \frac{\sqrt{T_{t_3}}}{a_3^* M_3^* \sqrt{T_{t_3}}}$$

$$D = \left( 1 - \frac{p_2}{p_3} \right) \frac{\gamma + 1}{2\gamma} \sqrt{\frac{T_{t_1}}{T_{t_0}}} \frac{1}{M_3^*} \left[ 1 - \frac{\gamma - 1}{\gamma + 1} M_3^{*2} \right]$$

For  $M_3^* \leq 1$  and  $M_2^* \leq M_{2cr}^*$ ,  $p_2 = p_3$ ; therefore  $D = 0$ .

For  $M_3^* = 1$  and  $M_2^* > M_{2cr}^*$ ,  $p_2 \neq p_3$ .

Therefore:

$$D = \left( 1 - \frac{p_2}{p_3} \right) \frac{\gamma + 1}{2\gamma} \sqrt{\frac{T_{t_1}}{T_{t_0}}} \left[ \frac{2}{\gamma + 1} \right] \neq 0$$

$$D = \frac{1}{\gamma} \sqrt{\frac{T_{t_1}}{T_{t_0}}} \left( 1 - \frac{p_2}{p_3} \right)$$

$$D = \frac{1}{\gamma} \sqrt{\frac{T_{t_1}}{T_{t_0}}} \left( 1 - \frac{\frac{p_2}{p_{t_0}}}{\frac{p_3}{p_{t_0}}} \right)$$



$$D = \frac{1}{\delta} \sqrt{\frac{T_{t1}}{T_{t0}}} \left( 1 - \frac{\frac{p_2}{P_{t0}}}{\left(\frac{p_3}{P_{t1}}\right)\left(\frac{P_{t1}}{P_{t0}}\right)} \right)$$

$$D = \frac{1}{\delta} \sqrt{\frac{T_{t1}}{T_{t0}}} \left\{ 1 - \left( \frac{1 - \frac{\delta-1}{\delta+1} M_2^{*2}}{\frac{2}{\delta+1}} \right)^{\frac{\gamma}{\delta-1}} \frac{P_{t0}}{P_{t1}} \right\}$$

Let

$$\xi = \frac{1}{\delta} \left\{ 1 - \left( \frac{1 - \frac{\delta-1}{\delta+1} M_2^{*2}}{\frac{2}{\delta+1}} \right)^{\frac{\gamma}{\delta-1}} \frac{P_{t0}}{P_{t1}} \right\}$$

$$D = \sqrt{\frac{T_{t1}}{T_{t0}}} \xi$$

Substituting into Equation (A-32), the dimensionless momentum equation is

$$\sqrt{\frac{T_{t1}}{T_{t0}}} M_3^* + \kappa M_2^* + \sqrt{\frac{T_{t1}}{T_{t0}}} \xi = (1 + \kappa) \sqrt{\frac{T_{t4}}{T_{t0}}} M_4^* \quad (\text{A-33})$$

By fixing  $P_{t5}/P_{t0}$ ,  $P_{t1}/P_{t0}$ ,  $T_{t1}/T_{t0}$ , and  $M_2^*$ , it is possible to obtain an explicit solution from the continuity, energy, and momentum equations since there remains only three unknowns  $\kappa$ ,  $T_{t4}/T_{t0}$ , and  $A_4/A_3$ .

The energy and momentum equations are solved simultaneously for  $\kappa$  and  $T_{t4}/T_{t0}$ . By eliminating  $T_{t4}/T_{t0}$  between the two equations, the following quadratic is obtained.

$$(M_4^{*2} - M_2^{*2}) \kappa^2 + \left\{ \left( \frac{T_{t1}}{T_{t0}} + 1 \right) M_4^{*2} - 2 \sqrt{\frac{T_{t1}}{T_{t0}}} M_2^* (M_3^* + \xi) \right\} \kappa + \frac{T_{t1}}{T_{t0}} \left\{ M_4^{*2} - (M_3^* + \xi)^2 \right\} = 0$$

This equation can be solved by using the quadratic formula

$$\gamma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

$$a = M_4^{*2} - M_2^{*2}$$

$$b = \left( \frac{T_{t1}}{T_{t0}} + 1 \right) M_4^{*2} - 2 \sqrt{\frac{T_{t1}}{T_{t0}}} M_2^* (M_3^* + \xi)$$

$$c = \frac{T_{t1}}{T_{t0}} \left\{ M_4^{*2} - (M_3^* + \xi)^2 \right\}$$

Once  $\gamma$  is determined, then  $T_{t4}/T_{t0}$  can be determined from the energy equation and  $A_4/A_3$  can be determined from the continuity equation. In addition, it is now possible to determine the area ratios  $A_2/A_3$ ,  $A_5/A_3$ , and  $A_6/A_3$ .

#### 4. Development of constant area theory:

This development was provided by Dr. Gawain in Reference 2. Basically, it is a process to non-dimensionalize the equations of continuity, energy and momentum. These equations are initially in the following form:

Continuity:

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_4 \quad (\text{A-34})$$

Energy:

$$\dot{m}_2 H_2 + \dot{m}_3 H_3 = \dot{m}_4 H_4 \quad (\text{A-35})$$

Momentum:

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \quad (\text{A-36})$$

First, consider the continuity equation (A-34). Expanding this equation gives the relationship:

$$\rho_2 A_2 M_2^* a_2^* + \rho_3 A_3 M_3^* a_3^* = \rho_4 A_4 M_4^* a_4^*$$

It must be understood that  $A_2 + A_3 = A_4$  for the constant area case.

The general relationship for the ratio of mass flow rate to the area is:

$$\dot{m}/A = \rho V = \rho a^* M^* = \frac{\rho}{\rho_t} \rho_t a^* M^* \quad (\text{A-37})$$

Using the isentropic relationship for the density ratio and expanding

$\rho_t a^*$  yields:

$$\dot{m}/A = \left\{ 1 - \frac{\gamma-1}{\gamma+1} M^{*2} \right\}^{\frac{1}{\gamma-1}} \frac{P_t}{g R T_t} \left\{ \frac{2\gamma}{\gamma+1} g R T_t \right\}^{\frac{1}{2}} M^*$$

Let

$$K_1 = \left\{ \frac{2\gamma}{(\gamma+1)gR} \right\}^{\frac{1}{2}}.$$

The final form of Equation (A-37) is:

$$\dot{m}/A = K_1 P_t (T_t)^{-\frac{1}{2}} \left( 1 - \frac{\gamma-1}{\gamma+1} M^{*2} \right)^{\frac{1}{\gamma-1}} M^*$$

It is convenient to define a velocity function  $\phi(M^*)$ :

$$\phi(M^*) \equiv \frac{\rho}{\rho_t} M^* \equiv \left( 1 - \frac{\gamma-1}{\gamma+1} M^{*2} \right)^{\frac{1}{\gamma-1}} M^*$$

Solving for the mass flow rate gives:

$$\dot{m} = K_1 A P_t (T_t)^{-\frac{1}{2}} \phi(M^*) \quad (\text{A-38})$$

Now substituting Equation (A-38) into Equation (A-34).

$$K_1 A_2 P_{t2} (T_{t2})^{-\frac{1}{2}} \phi(M_2^*) + K_1 A_3 P_{t3} (T_{t3})^{-\frac{1}{2}} \phi(M_3^*) = K_1 A_4 P_{t4} (T_{t4})^{-\frac{1}{2}} \phi(M_4^*)$$

It can be seen from Figure 2 that the total state conditions at stations

1 and 3, and stations 0 and 2 are equal. The preceding equation is

non-dimensionalized by dividing through by  $K_1 A_3 P_{t0} (T_{t0})^{-\frac{1}{2}}$ . This

results in the final non-dimensional form of the continuity equation.



$$\gamma \phi(M_2^*) + \frac{P_{t1}}{P_{t0}} \sqrt{\frac{T_{t0}}{T_{t1}}} \phi(M_3^*) = (1 + \gamma) \frac{P_{t4}}{P_{t0}} \sqrt{\frac{T_{t0}}{T_{t4}}} \phi(M_4^*) \quad (A-39)$$

The energy equation (A-35) for the constant area jet pump can be expanded to yield the relationship:

$$\rho_2 A_2 V_2 c_p T_{t2} + \rho_3 A_3 V_3 c_p T_{t3} = \rho_4 A_4 V_4 c_p T_{t4} \quad (A-40)$$

In general, the term  $\rho V T_t$  can be expanded into the form:

$$\rho V T_t = \rho \frac{c}{c_t} a^* M^* T_t = T_t \rho_t a^* \phi(M^*)$$

and,

$$T_t \rho_t a^* = \left\{ \frac{2\gamma}{(\gamma+1)R_g} \right\}^{1/2} P_t (T_t)^{1/2} = K_1 P_t (T_t)^{1/2}$$

Therefore;

$$\rho V T_t = K_1 P_t (T_t)^{1/2} \phi(M^*)$$

Substituting this relationship into Equation (A-40) and non-dimensionalizing in the same manner as before results in the final form of the energy equation.

$$\gamma \phi(M_2^*) + \frac{P_{t1}}{P_{t0}} \sqrt{\frac{T_{t1}}{T_{t0}}} \phi(M_3^*) = (1 + \gamma) \frac{P_{t4}}{P_{t0}} \sqrt{\frac{T_{t4}}{T_{t0}}} \phi(M_4^*) \quad (A-41)$$

The specific heat at constant pressure ( $c_p$ ) is assumed to be constant.

Finally, considering the momentum equation (assuming steady, frictionless flow and the velocities are uniform at stations 2, 3, and 4):

$$\dot{m}_4 V_4 - \dot{m}_2 V_2 - \dot{m}_3 V_3 = p_2 A_2 + p_3 A_3 - p_4 A_4 \quad (A-36)$$

Rearranging and substituting for  $\dot{m}$  and  $A_4$  gives;

$$A_2(p_2 + \rho_2 V_2^2) + A_3(p_3 + \rho_3 V_3^2) = (A_2 + A_3)(p_4 + \rho_4 V_4^2)$$

Now consider  $(p + \rho V^2)$  in general. This factor can be expanded to yield,

$$p + \rho V^2 = p + \frac{p}{R_g T} a^{*2} M^{*2} = p \left( 1 + \frac{2\gamma}{\gamma+1} \frac{T_t}{T} M^{*2} \right)$$

where

$$\frac{T_t}{T} = \left( 1 + \frac{\gamma-1}{\gamma+1} M^{*2} \right)^{-1}$$

and

$$\frac{p}{p_t} = \left( 1 + \frac{\gamma-1}{\gamma+1} M^{*2} \right)^{\frac{\gamma}{\gamma-1}}$$

Therefore;

$$p + \rho V^2 = \frac{p}{p_t} p_t \left\{ \frac{1 + M^{*2}}{1 + \frac{\gamma-1}{\gamma+1} M^{*2}} \right\} = p_t \left\{ 1 + \frac{\gamma-1}{\gamma+1} M^{*2} \right\}^{\frac{1}{\gamma-1}} \left\{ 1 + M^{*2} \right\}$$

It is helpful to define  $\psi(M^*)$  as;

$$\psi(M^*) \equiv \left\{ 1 + \frac{\gamma-1}{\gamma+1} M^{*2} \right\}^{\frac{1}{\gamma-1}} \left\{ 1 + M^{*2} \right\}$$

So the final form of the term  $(p + \rho V^2)$  is:

$$p + \rho V^2 = p_t \psi(M^*) \quad (\text{A-42})$$

Substituting Equation (A-42) into Equation (A-36) and non-dimensionalizing yields the momentum equation:

$$\gamma \psi(M_2^*) + \frac{p_{t1}}{p_{t0}} \psi(M_3^*) = (1 + \gamma) \frac{p_{t4}}{p_{t0}} \psi(M_4^*) \quad (\text{A-43})$$

From the continuity equation, the mass flow ratio is determined to be;

$$\dot{m}_2 / \dot{m}_3 = \chi = \gamma \frac{p_{t0}}{p_{t1}} \sqrt{\frac{T_{t1}}{T_{t0}}} \frac{\phi(M_2^*)}{\phi(M_3^*)} \quad (\text{A-44})$$

And from the energy equation, the mixing tube discharge temperature ratio is:

$$\frac{T_{t4}}{T_{t0}} = \frac{\gamma + T_{t1}/T_{t0}}{\gamma + 1} \quad (\text{A-45})$$

Dividing Equation (A-41) by Equation (A-43) results in a quadratic relationship in terms of  $M_4^*$ .

$$\frac{\phi(M_4^*)}{\psi(M_4^*)} = \sqrt{\frac{T_{t0}}{T_{t4}}} \left\{ \frac{\gamma \phi(M_2^*) + \frac{P_{t1}}{P_{t0}} \sqrt{\frac{T_{t1}}{T_{t0}}} \phi(M_3^*)}{\gamma \psi(M_2^*) + \frac{P_{t1}}{P_{t0}}} \psi(M_3^*)} \right\} \quad (\text{A-46})$$

where,

$$\frac{\phi(M_4^*)}{\psi(M_4^*)} = \frac{M_4^*}{1 + M_4^{*2}}$$

It can be shown that the right side of Equation (A-46) must be equal to or less than one-half. Let,

$$f(M_4^*) = \frac{M_4^*}{1 + M_4^{*2}} \quad (\text{A-47})$$

Taking the derivative of this function with respect to  $M_4^*$ .

$$\frac{df(M_4^*)}{dM_4^*} = \frac{1 - M_4^{*2}}{(1 + M_4^{*2})^2}$$

Setting the derivative equal to zero and solving for  $M_4^*$  yields,

$$M_4^{*2} = 1$$

Choosing the positive root and substituting into Equation (A-47) gives the maximum value of the function  $f(M_4^*)$ .

$$f(M_4^*)_{\max} = 1/2$$



Therefore;

$$f(M_4^*) \leq 1/2$$

This condition is a limit for the constant area case.

Solutions for optimum constant area jet pumps are contained in Appendix C. An iterative procedure was used to satisfy the boundary condition that  $P_{t4}/P_{t0}$  must equal  $P_{t5}/P_{t0}$ . This was accomplished by varying the area ratio  $A_2/A_3$  until  $P_{t4}/P_{t0}$  was equal to the specified value of  $P_{t5}/P_{t0}$ . It is emphasized that each line of the computer results represents the optimum pump of a family of constant area jet pumps.



## APPENDIX B

### Itemized cost estimate

The following pages show the itemized cost estimate for various test facility configurations. The numbers for each item up to item 22 corresponds to the numbers shown on Figures 26 through 34. The labor estimate was made by Mr. Robert Besel, Laboratory Supervisor for the Aeronautical Engineering Department.

Material estimates were solicited from numerous companies. The code used for denoting the company is as follows:

Code	Company
MC	McMaster-Carr Supply Co.
DU	Ducommun Metals and Supply Co.
RE	Reliance Steel Co.
ME	Meriam Instrument Co.
PA	Pacific Metals Co.
RY	Ryerson Steel
GC	General Controls Corp.
USC	United Sensor and Control Corp.

The total cost estimates are included in the following itemized cost estimates.





ITEM	Units	Unit Price	Maximum Price	Minimum Price
1. Pressure tank: Stock No. - 437P611(MC)	1		149.60	149.60
2. Flange: ASA 150 Forged Steel Slip-on flange, NPS-4 in. (DU)	3	20.00	60.00	60.00
3. Pipe: ASTM A-53 SCH 40 4 inch pipe (RE)	9 ft 15 ft	1.80/ft	18.20	36.20
4. Orifice plates and flanges: (ME) 4 in. pipe-Series 30 flange orifice plate 6 in. pipe-Series 30 flange orifice plate	1 1 1 1		32.00 10.00 50.50 11.25	32.00 10.00 50.50 11.25
5. Elbow: 90° radius elbow for 4 in. pipe, No. 406 (DU)	1		19.50	19.50
6. Pipe: Type 304 SCH 40 Stainless steel pipe (PA) 6 inch pipe 5 inch pipe 4 inch pipe	8 ft 10 ft 2 ft 8 ft	39.44/ft 35.53/ft 35.53/ft 26.22/ft	316.60 355.98 71.06 210.27	
7. Flange: Type 316 MSS Stainless steel slip-on flanges (DU) NPS-4 inch NPS-5 inch NPS-6 inch	6 8 2 6	21.37 33.69 33.69 36.97	128.22 268.92 67.38 221.82	





ITEM	Units	Unit Price	Maximum Price	Minimum Price
8. Nozzle: Type 304 Stainless steel round bar, 6 in. dia. Spec. QQ-S-763B (PA)	144 lb 46 lb	.87/lb .87/lb	128.58	42.86
9. Annulus - Hot rolled steel ring, O.D. 10" (RY) I.D. 4 in-3 in. thick I.D. 5 in-2½ in. thick I.D. 6 in-2in. thick	1 1 1		63.08 45.06 36.05	63.08
10. Backing Ring - ASA 150 lb forged steel blind flange NPS - 10 inches (DU)	1		58.00	
11. Sheet: cold rolled 12 gage steel (48" x 120") (PA)	175 lb	.22/lb	39.20	39.20
12. Flange: ASA 150 forged steel slip-on flange, (DU) NPS - 6 in. NPS - 4 in.	2 4	27.00 20.00	54.00	54.00 80.00
13. Pipe: ASTM-A-53, SCH 40, 6 inch steel pipe (RE)	12 ft	2.85/ft	36.20	36.20
14. Flange: Type 316 MSS stainless steel slip-on "special order" O.D. 16". NPS 4,5,6 (DU)	3 1	90.38	271.14	90.38
15. Flange: ASA 150 forged steel slip-on reducing flange (DU) 4 x 11" O.D. 5 x 11" O.D. 6 x 11" O.D.	1 1 2		49.00 49.00 98.00	



ITEM	Units	Unit Price	Maximum Price	Minimum Price
16. Tubing: AISI MT-1015, 10 Gage dia. 2.375", length 24' (RE)	77 lb	.67/lb	52.59	52.59
17. Turn Buckles: Drop forged Type 302 (DU)	6	1.10	6.60	6.60
18. Channel: Aluminum 6063-T5 Die #2388, Length 48' (PA)	20 lb	1.66/lb	32.20	
19. Bar: Aluminum 2024-T4 rectangular bar, $\frac{1}{2}$ " x 2" x 36" (PA)	21.5 lb	2.08/lb	44.72	
20. Casters: SB-4689 BASSICK (DU)	8	6.20	49.60	
21. Mounting plates: Mixing tube actuator mounts (local)	48 8	1.00	48.00	8.00
22. Valve: Electrically actuated remote controlled butterfly valve (GC)	1		326.10	326.10
23. Tubing: Copper tubing, O.D. $\frac{1}{2}$ " 50' coil (DU)	4	6.06	24.24	
24. Tubing: Imperial poly-flo tubing, Stock #66-P (DU)	200 ft	.09/ft	17.80	
25. Wall Taps: static pressure mixing tube wall taps (DU)	72	.50	36.00	





ITEM	Units	Unit Price	Maximum Price	Minimum Price
26. Connectors: wall tap to copper tube connectors, No. 768 - FS (DU)	72	1.89	136.08	
27. Connectors: copper tube to plastic tube connectors, No. 262-PC (DU)	72	0.26	18.73	
28. Plugs: Dryseal threaded steel pressure plugs (DU)	50	.15	7.60	
29. Probe: Kiel - temperature probe, Order # KT-14-C-ca-B (USC)	3	30.00	90.00	90.00
30. Receiver: USC - 1396-6	2	380.00	760.00	
31. Mechanical Cables: USC-1103-30	2	68.00	136.00	
32. Collets: USC-1499-125	2	1.50	3.00	3.00
Proposal For Remote Electrically Controlled System				
33. Transmitter: USC-125I	1		310.00	
34. Motor Box: USC-1462-B	1		675.00	
Total Material Cost For Remote Electrical Control System			5555.42	





ITEM	Units	Unit Price	Maximum Price	Minimum Price
Alternate Proposal For Remote Manual Controlled System				
35. Transmitter: USC-1793 replaces items 33 and 34	1		220.00	
Total Material Cost For Remote Manual Control			4787.42	
Instrumentation Necessary For Minimum Cost System				
36. Manual traverse unit: replaces items 30, 31, 33, 34, and 35 USC-1108-12	2	72.00		144.00
Total Cost For Minimum Priced System				1601.50

### Labor Estimate

	Man Hours
1. Drill and tap flanges for primary system	7
2. Install mass flow orifice plates and flanges	7
3. Fabrication of nozzles	90
4. Fabrication of mixing tube sections	108
5. Fabrication of bell-mouth annulus	48
6. Fabrication of secondary air plenum	32
7. Drill and tap flanges for secondary system	2
8. Install butterfly valve	4
9. Fabricate mixing tube stanchions	6
10. Fabricate handling cart	16
11. Assemble complete system and hook-up instrumentation	42
 Total estimated man hours	 362

## APPENDIX C

### Computer Results

This appendix contains computer solutions for a set of optimum constant area and constant pressure heated jet pumps for the conditions specified. Each line of data represents the parameters defining one optimum pump. The momentum coefficient is constant for each page of data.

Also included is a sample computer program and solutions for one jet pump with a fixed area ratio,  $A_2/A_3 = 2.16$ , and fixed supply conditions. Each line represents the theoretical performance for the conditions specified.

The symbols used in the computer programs are as follows:

PT5PO = the ratio of the demanded discharge total pressure to the ambient pressure.

PnPm = pressure ratio, station (n) to (m).

TnTm = temperature ratio, station (n) to (m).

AnAm = area ratio, station (n) to (m).

SACHn = velocity ratio ( $M^*$ ) at station (n).

TT5TO = ratio of the discharge total temperature to the ambient temperature.

WMWJ = mass flow ratio, secondary to primary.

G =  $\gamma$ , ratio of specific heats = 1.4.

R = rho, density.





```

CMX = CA*{1,*MMWJ}*SACH5*TT5TO*.5
CMP = CMX/((PIPO**CD)-1.)
CPM = 1./CMP
CMQ = CMX/((TTTO-(PIPO**CD))
COM = 1./CMQ
OPRINT 90, PIPO,SACH2,SACH3,SACH4,A2A3,A5A4,A3A6,
1A4A6,TTTO,PTSP0,MMWJ,CPM,CM
90 FORMAT(1F9.4,1F9.4,1F9.4,4X
40 PRINT 63
63 FORMAT(////)
30 TTTO = XTTO*.1*KK
20 PTSP0 = XPTSP0*.1*XJ
62 PRINT 62
62 FORMAT (1H1)
END

```

```

1.10 1.0 1.2 0.251 1.4

```

83  
84  
85  
86  
87  
88  
89  
90  
91  
92  
93  
94

PT5P0	A5A6	SACW5	CWA	CWA										CPH	COM
				SACW5	SACW3	SACW4	A2A3	A5A4	A3A6	A4A6	T1T0	TT5T0	WMWJ		
1.100	1.690	.401	.442												
PIP0	SACW2	SACW3	SACW4	A2A3	A5A4	A3A6	A4A6	T1T0	TT5T0	WMWJ	CPH	COM			
1.2000	.9610	1.0000	1.0295	1.0012	1.6885	.5002	1.0010	1.0000	1.0000	.8328	.4360	-.4360			
1.6000	.9650	1.0000	1.0331	5.0047	1.6880	.1667	1.0013	1.0000	1.0000	3.1233	.5209	-.5209			
2.0000	.9660	1.0000	1.0341	9.0079	1.6879	.1000	1.0014	1.0000	1.0000	4.4977	.5954	-.5954			
2.4000	.9660	1.0000	1.0341	13.2115	1.6879	.0715	1.0014	1.0000	1.0000	5.4139	.6622	-.6622			
2.8000	.9660	1.0000	1.0341	17.0151	1.6879	.0556	1.0014	1.0000	1.0000	6.0684	.7231	-.7231			
1.2000	.5390	.7618	.6663	.9364	1.4659	.5954	1.1531	2.0000	1.5293	.8895	.3420	6.0541			
1.6000	.7820	1.0000	.8696	4.3903	1.6556	.1894	1.0209	2.0000	1.2147	3.6587	.4183	2.4924			
2.0000	.8280	1.0000	.9101	8.0381	1.6738	.1117	1.0098	2.0000	1.1543	5.4813	.4702	1.6762			
2.4000	.8460	1.0000	.9261	11.7441	1.6791	.0790	1.0066	2.0000	1.1295	6.7226	.5175	1.3034			
2.8000	.8570	1.0000	.9359	15.4619	1.6819	.0610	1.0050	2.0000	1.1160	7.6170	.5615	1.0802			
1.2000	.3860	.6678	.5533	.9360	1.2943	.6744	1.3060	3.0000	2.0585	.8895	.2948	10.7310			
1.6000	.5960	1.0000	.7119	4.2089	1.5224	.2131	1.1103	3.0000	1.4276	3.6775	.3943	4.9639			
2.0000	.6740	1.0000	.7767	7.4430	1.5889	.1260	1.0638	3.0000	1.3017	5.6287	.4328	3.5193			
2.4000	.7090	1.0000	.8064	10.8044	1.6140	.0887	1.0742	3.0000	1.2498	7.0073	.4745	2.8646			
2.8000	.7280	1.0000	.8228	14.2227	1.6263	.0683	1.0393	3.0000	1.2218	8.0179	.5128	2.4858			
1.2000	.3170	.6324	.5089	.9347	1.2151	.7184	1.3910	4.0000	2.5878	.8894	.2629	14.4878			
1.6000	.4920	.9827	.6299	4.2005	1.4154	.2295	1.1942	4.0000	1.6413	3.6776	.3584	7.1230			
2.0000	.5720	1.0000	.6925	7.2198	1.4993	.1371	1.1274	4.0000	1.4503	5.6623	.4080	5.1801			
2.4000	.6120	1.0000	.7250	10.3662	1.5371	.0967	1.1498	4.0000	1.3704	7.0986	.4480	4.2811			
2.8000	.6350	1.0000	.7440	13.5812	1.5574	.0744	1.0853	4.0000	1.3273	8.1659	.4840	3.7617			
1.2000	.2750	.6135	.4845	.9348	1.1692	.7466	1.4456	5.0000	3.1170	.8895	.2396	17.6807			
1.6000	.4080	.9561	.5826	4.2021	1.3430	.2419	1.2586	5.0000	1.8551	3.6777	.3371	9.0457			
2.0000	.5030	1.0000	.6383	7.1231	1.4275	.1457	1.1841	5.0000	1.5994	5.6778	.3878	6.6950			
2.4000	.5430	1.0000	.6694	10.1453	1.4700	.1031	1.1498	5.0000	1.4703	7.1381	.4274	5.5874			
2.8000	.5680	1.0000	.6893	13.2171	1.4953	.0794	1.1303	5.0000	1.4331	8.2368	.4623	4.9440			



CONSTANT PRESSURE HEATED JETPUMP, OPTIMUM SOLUTIONS

PTSP0	ASA6	SACH5	CMA	ASA6	A2A3	ASA4	A3A6	A4A6	T1T0	TTST0	WMWJ	CPH	CPM
1.200	1.309	.552	.662	SACH4									
P1P0	SACH2	SACH3											
1.6000	.9280	1.0000	1.0593	2.0080	1.3030	.3338	1.0042	1.0000	1.0000	1.0000	1.2472	.6954	-.6954
2.0000	.9310	1.0000	1.0618	4.0147	1.3025	.2003	1.0046	1.0000	1.0000	1.0000	1.9958	.7948	-.7948
2.4000	.9320	1.0000	1.0626	6.0214	1.3024	.1431	1.0047	1.0000	1.0000	1.0000	2.4950	.8841	-.8841
2.8000	.9330	1.0000	1.0634	8.0276	1.3022	.1113	1.0048	1.0000	1.0000	1.0000	2.8515	.9655	-.9655
1.6000	.7700	1.0000	.9313	1.7719	1.3011	.3627	1.0057	2.0000	1.4054	1.4054	1.4666	.5344	3.1837
2.0000	.8060	1.0000	.9598	3.6125	1.3060	.2172	1.0019	2.0000	1.2908	1.2908	2.4387	.6095	2.1734
2.4000	.8220	1.0000	.9726	5.4774	1.3073	.1545	1.0009	2.0000	1.2436	1.2436	3.1044	.6751	1.7003
2.8000	.8310	1.0000	.9798	7.3519	1.3079	.1198	1.0005	2.0000	1.2181	1.2181	3.5855	.7348	1.4135
1.6000	.5950	1.0000	.8004	1.6908	1.2458	.3903	1.0504	3.0000	1.8079	1.8079	1.4756	.4694	6.0632
2.0000	.6570	1.0000	.8511	3.3450	1.2735	.2363	1.0275	3.0000	1.5699	1.5699	2.5094	.5415	4.4036
2.4000	.6940	1.0000	.8728	5.2587	1.2830	.1683	1.0199	3.0000	1.4714	1.4714	3.2423	.6004	3.6251
2.8000	.7110	1.0000	.8857	6.7912	1.2879	.1304	1.0160	3.0000	1.4183	1.4183	3.7810	.6531	3.1659
1.6000	.4920	.9827	.7310	1.6854	1.1950	.4076	1.0950	4.0000	2.2118	2.2118	1.4757	.4244	8.4345
2.0000	.5670	1.0000	.7808	3.2415	1.2329	.2501	1.0613	4.0000	1.8507	1.8507	2.5263	.4964	6.3027
2.4000	.6020	1.0000	.8053	4.8549	1.2488	.1789	1.0478	4.0000	1.6997	1.6997	3.2878	.5528	5.2821
2.8000	.6230	1.0000	.8204	6.4909	1.2577	.1386	1.0404	4.0000	1.6179	1.6179	3.8549	.6022	4.6797
1.6000	.4280	.9561	.6917	1.6861	1.1599	.4198	1.1281	5.0000	2.6157	2.6157	1.4757	.3903	10.4713
2.0000	.5010	1.0000	.7368	3.1890	1.1998	.2601	1.0906	5.0000	2.1324	2.1324	2.5322	.4616	7.9697
2.4000	.5370	1.0000	.7604	4.7406	.2183	.1869	1.0740	5.0000	1.9285	1.9285	3.3079	.5165	6.7530
2.8000	.5580	1.0000	.7747	6.3238	1.2287	.1453	1.0650	5.0000	1.8179	1.8179	3.8907	.5639	6.0312







```

11 SACH3=1.0
13 P3P0=P1P0*(CG**CJ)
1 P2P=P2P0/P3P0
22P0=P2P0**CJ
1311=(1.-CA*SACH3*SACH3)
23P=P4P0/(11P0*311)
13 CONTINUE
WWJ=A2A3**22P0*SACH2/(P4P0**SACH3)
114T=(WWJ+11P0)/(P4P0+1.0)
D=RCY*(1.-2P0*(1.-CA*(SACH4*SACH3)))/SACH3
XEL=(P2P0*A2A3*SACH2+P3P0**SACH3)/(114T0**5)/(A2A3+1.)
XBE=((A2A3+1.)**XBE*(1.-CA*(SACH4**2.)))/(P3P0*SACH3)
XEP=(WWJ*SACH2+P3P0*SACH3+P2P0*XBE)/XBE
XLA=((A2A3+1.)*(11P0**5))/XBE
XAB=XEP*XEP*(XLA**XLA)
XAB=XAB/XAB
1F(ABS(F(ARAD))-0.005) 36,56,55
56 ARAD=0.0
55 CONTINUE
1F (ARAD) 33,57,57
57 CONTINUE
SACH4=(-XEP+(ARAD**5))/(2*(XLA**CA-XLA))
1F (SACH4) 53,53,54
54 CONTINUE
P4P0=XEP-SACH4*XLA
PT4P=P4P0/(1.-CA*SACH4*SACH4)**CB
15 GO TO (1,2,3),NUM3
1 1F(PR-PT4P)16,3,17
2 1F(PR-PT4P)18,3,17
16 A2A3=A2A3
PT4P0=PT4P0
A2A3=A2A3+1.0
GO TO 13
17 NUM3=2
A2A3=A2A3
PT4P0=PT4P0
A2A3=A2A3-.1
GO TO 13
19 NUM3=4
A2A3=(A2A3*(PT4P0-PR)-A2A3*(PT4P0-PR))/(PT4P0-PT4P0)
GO TO 13
3 1F(WWJ-WWJ0)4,76,76
76 WWJ0=WWJ
53 CONTINUE
SA=4T=SACH2
SACH4T=SACH4
A2A3T=A2A3
P4P0T=P4P0
PT4P0T=PT4P0
WWJT=WWJ0
114T0T=114T0
50 SACH2=XSACH2+.002*XN
33 A5A6=(P4P0T**CK)*SACH4T/SACH5
A6A6=((1.-CA*(SACH4T**2.))/CG)**CJ)*SACH4T
A4A3=A2A3T+1.
A2A6=A2A3T/(A4A3*A6A6)

```



CONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTIONS

PT5P0	AS46	SACH5	CMA	PT4P0	T1TJ	TT3TJ	MMWJ	CPM	CUM
1.100	1.590	.401	.442						
PIPO	SACH2	SACH3	SACH4	P4P0	T1TJ	TT3TJ	MMWJ	CPM	CUM
* 1.200	1.300	1.000	1.300	.581	1.100	1.000	.833	.436	-4.436
* 1.300	1.300	1.000	1.000	.581	1.100	1.000	1.538	.458	-4.458
* 1.400	1.300	1.000	1.000	.581	1.100	1.000	2.143	.480	-4.480
* 1.500	1.300	1.000	1.000	.581	1.100	1.000	2.667	.501	-4.501
* 1.600	1.300	1.000	1.000	.581	1.100	1.000	3.125	.521	-4.521
* 1.800	1.300	1.000	1.000	.581	1.100	1.000	3.688	.559	-4.559
* 2.000	1.300	1.000	1.000	.581	1.100	1.000	4.250	.595	-4.595
* 2.400	1.300	1.000	1.000	.581	1.100	1.000	5.417	.662	-4.662
* 2.800	1.300	1.000	1.000	.581	1.100	1.000	6.071	.723	-4.723
1.200	.538	.761	.665	.841	1.100	1.529	.891	.342	6.251
1.300	.532	.897	.741	.796	1.100	1.372	1.689	.369	4.376
1.400	.712	1.000	.810	.734	1.100	1.293	2.413	.389	3.463
1.500	.754	1.000	.845	.706	1.100	1.246	3.169	.404	2.887
* 1.600	.760	1.000	.841	.709	1.100	1.215	3.656	.419	2.494
* 1.800	.754	1.000	.820	.726	1.100	1.177	4.638	.447	1.996
* 2.000	.750	1.000	.808	.735	1.100	1.156	5.423	.474	1.690
* 2.400	.744	1.000	.792	.747	1.100	1.132	6.598	.525	1.324
* 2.800	.740	1.000	.783	.754	1.100	1.119	7.434	.573	1.102
1.200	.386	.668	.553	.916	1.100	2.058	.891	.295	10.726
1.300	.456	.791	.602	.884	1.100	1.744	1.689	.328	8.990
1.400	.516	.890	.650	.852	1.100	1.586	2.413	.351	6.644
1.500	.560	.967	.684	.828	1.100	1.491	3.072	.369	5.641
1.600	.594	1.000	.710	.809	1.100	1.428	3.678	.384	4.964
* 1.800	.536	1.000	.741	.786	1.100	1.348	4.740	.410	4.474
* 2.000	.532	1.000	.723	.799	1.100	1.303	5.607	.434	3.930
* 2.400	.524	1.000	.700	.817	1.100	1.253	6.898	.480	2.900
* 2.800	.518	1.000	.685	.827	1.100	1.227	7.810	.524	2.559
1.200	.316	.632	.508	.943	1.100	2.587	.891	.263	14.482
1.300	.374	.750	.544	.921	1.100	2.116	1.689	.297	11.166
1.400	.424	.845	.581	.898	1.100	1.879	2.413	.322	9.462
1.500	.462	.920	.609	.880	1.100	1.737	3.072	.342	8.012
1.600	.490	.982	.628	.867	1.100	1.641	3.678	.358	7.123
1.800	.438	1.000	.665	.842	1.100	1.522	4.751	.385	5.934
* 2.000	.456	1.000	.672	.837	1.100	1.451	5.659	.408	5.183
* 2.400	.448	1.000	.644	.856	1.100	1.374	7.019	.452	4.318
* 2.800	.442	1.000	.627	.868	1.100	1.334	7.976	.493	3.831
1.200	.274	.613	.484	.957	1.100	3.116	.891	.239	17.673
1.300	.326	.729	.514	.939	1.100	2.488	1.689	.274	13.821
1.400	.370	.821	.545	.921	1.100	2.172	2.412	.300	11.366
1.500	.402	.895	.564	.908	1.100	1.982	3.072	.320	10.106
1.600	.428	.956	.583	.897	1.100	1.855	3.678	.337	9.046
1.800	.474	1.000	.618	.874	1.100	1.695	4.754	.365	7.615
2.000	.504	1.000	.640	.859	1.100	1.599	5.674	.388	6.895
* 2.400	.494	1.000	.605	.882	1.100	1.495	7.079	.430	5.622
* 2.800	.488	1.000	.587	.895	1.100	1.441	8.065	.470	5.023

STAR INDICATES SACH4 MAXIMUM



CONSTANT AREA HEATED JETPUMP, OPTIMUM SOLUTIONS

	PT5P0	45A6	SACH5	CMA	SACH4	SACH3	SACH2	SACH1	A2A3	A3A4	A3A6	PIP0	PT4PC	TIT	TT5T0	MM0J	CPM	CuM
	1.200	1.309	.552	.662	1.200	1.000	1.000	1.000	.500	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.300	1.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.400	1.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	1.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	2.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	3.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	4.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	5.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	6.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	7.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.309	.667	.634	1.200	1.000	1.200	.335	.611	-.611
	8.000	1.000	1.000	1.000	1.000	1.000	1.000											

STAR INDICATES SACH4 MAXIMUM

CONSTANT ARCA HEATED JETPUMP, OPTIMUM SOLUTIONS

PTSP0	ASA6	SACH5	GMA	A2A3	ASA4	A3A6	P4P0	PT4P0	TTTJ	TTJTC	WMWJ	CPM	CWH
1.300	1.162	.658	.856										
PIPO	SACH2	SACH3	SACH4										
* 1.400	1.000	1.000	1.000	.335	1.161	.749	.687	1.500	1.000	1.000	.259	.742	-.742
* 1.500	1.000	1.000	1.000	.668	1.161	.600	.687	1.500	1.000	1.000	.445	.775	-.775
* 1.600	1.000	1.000	1.000	1.000	1.162	.500	.687	1.500	1.000	1.000	.625	.806	-.806
* 1.800	1.000	1.000	1.000	1.668	1.161	.375	.687	1.500	1.000	1.000	.926	.865	-.865
* 2.000	1.000	1.000	1.000	2.334	1.161	.300	.687	1.500	1.000	1.000	1.167	.921	-.921
* 2.400	1.000	1.000	1.000	3.667	1.162	.214	.687	1.500	1.000	1.000	1.528	1.025	-1.025
* 2.800	1.000	1.000	1.000	5.000	1.162	.167	.687	1.500	1.000	1.000	1.786	1.119	-1.119
* 1.400	.586	.989	.921	.302	1.153	.774	.763	1.500	2.000	1.788	.269	.342	4.829
* 1.500	.738	1.000	.966	.594	1.160	.628	.719	1.500	2.000	1.661	.514	.574	4.598
* 1.600	.750	1.000	.964	.898	1.160	.528	.721	1.500	2.000	1.577	.734	.602	3.584
* 1.800	.754	1.000	.928	1.525	1.154	.399	.756	1.500	2.000	1.474	1.111	.650	2.926
* 2.000	.750	1.000	.899	2.163	1.147	.320	.784	1.500	2.000	1.414	1.415	.695	2.479
* 2.400	.744	1.000	.867	3.448	1.137	.230	.814	1.500	2.000	1.348	1.872	.777	1.957
* 2.800	.740	1.000	.650	4.737	1.130	.179	.830	1.500	2.000	1.313	2.199	.851	1.637
* 1.400	.490	.876	.800	.306	1.106	.804	.876	1.500	3.000	2.576	.269	.452	8.497
* 1.500	.570	.972	.867	.570	1.136	.651	.814	1.500	3.000	2.319	.516	.485	7.412
* 1.600	.596	1.000	.876	.848	1.140	.551	.805	1.500	3.000	2.149	.741	.513	6.629
* 1.800	.630	1.000	.893	1.410	1.145	.421	.789	1.500	3.000	1.936	1.166	.561	5.372
* 2.000	.652	1.000	.862	2.516	1.135	.359	.818	1.500	3.000	1.811	1.465	.602	4.895
* 2.400	.624	1.000	.813	3.266	1.113	.245	.864	1.500	3.000	1.675	1.922	.676	4.060
* 2.800	.618	1.000	.786	4.526	1.097	.192	.889	1.500	3.000	1.605	2.516	.743	3.600
* 1.400	.402	.835	.753	.308	1.077	.825	.918	1.500	4.000	3.363	.269	.395	11.551
* 1.500	.470	.924	.808	.569	1.110	.667	.869	1.500	4.000	2.979	.516	.428	10.023
* 1.600	.492	.963	.812	.847	1.112	.566	.865	1.500	4.000	2.723	.741	.456	9.061
* 1.800	.540	1.000	.846	1.364	1.128	.435	.834	1.500	4.000	2.402	1.140	.503	7.742
* 2.000	.556	1.000	.841	1.925	1.126	.353	.838	1.500	4.000	2.210	1.479	.542	6.878
* 2.400	.548	1.000	.781	3.154	1.095	.255	.893	1.500	4.000	2.001	1.998	.611	5.037
* 2.800	.542	1.000	.749	4.396	1.074	.201	.922	1.500	4.000	1.891	2.368	.673	5.231
* 1.400	.350	.813	.729	.308	1.060	.838	.939	1.500	5.000	4.151	.269	.356	13.741
* 1.500	.410	.898	.777	.567	1.092	.679	.896	1.500	5.000	3.659	.516	.387	12.421
* 1.600	.430	.957	.780	.844	1.094	.576	.894	1.500	5.000	3.297	.741	.414	11.117
* 1.800	.474	1.000	.807	1.352	1.109	.445	.970	1.500	5.000	2.868	1.141	.460	9.995
* 2.000	.500	1.000	.822	1.871	1.117	.362	.856	1.500	5.000	2.611	1.483	.498	8.289
* 2.400	.494	1.000	.760	3.080	1.081	.263	.912	1.500	5.000	2.326	2.016	.563	7.363
* 2.800	.488	1.000	.724	4.313	1.055	.207	.944	1.500	5.000	2.178	2.396	.622	6.655

STAR INDICATES SACH4 MAXIMUM

..J0R0U3IF, SUMMERS AND RIDDER TEN MIN MAX

```

PROGRAM JETPUMP
C  CONSTANT AREA HEATED JETPUMP, VARIABLE BACK PRESSURE
5  FORMAT(5F10.3)
   G=CAMWA
   CA=(G-1.)/(G+1.)
   CB=(G-1.)/G
   CE=(G+1.)/(G-1.)
   CH=2.*G/(G+1.)
   CJ=1./(G-1.)
   CY=(G+1.)/(2.*G)
   PIPO=XPIPO
   OJ40 M=1.7
   XM=M
   TIPO=XTIPO
   XK=X
   PRINT 61
61  FORMAT(1H,///30X,5BHFIKEO AREA HEATED JET PUMP, VARIABLE DISCHARG
      LE PRESSURE
      ///)
60  PRINT 62 X,WHPIPO UX,WHTIPO UX, 4A2A3
70  PRINT 70,PIPO,TIPO,A2A3
80  PRINT 80
80  FORMAT(1H,///30X,5BHFIKEO AREA HEATED JET PUMP, VARIABLE DISCHARG
      LE PRESSURE
      ///)
26  CONTINUE
   WHMWJ UX, WHCPM UX, WHCOM UX, WHGMA /)
   P2PO=(1.-CA*(SACH2**2))**CB
   SACH3=(CE*(1.-((P2PO*PPIPO)**CD))**5
   P2PO=X2PO
   SACH3=1.110,11,11
11  SACH3=1.110
10  P2PO=XPIPO*(CG**CB)
   P2PO=X2PO/P3PO
   R2PO=X2PO**CB
   R3PO=X3PO**CB
   WHMWJ UX, WHCPM UX, WHCOM UX, WHGMA /)
   TTITO=(WHMWJ*TIPO)/(WHMWJ*1.0)
   O=B*CA*(1.-P2PO)**(1.-((CA*(SACH3**SACH3)))/(SACH3
   XAL=(R2PO*A2A3*SACH2**R3PO**SACH3)/(A2A3*1.)
   XE=(A2A3*1.-P2PO)**CB
   XLA=(WHMWJ*1.0)/(TIPO**5))**XBE
   XRAO=XEP*XP*(XAL*CA-XLA)**XAL
   ARAO=XRAO
   IF (ABS(F(ARAO)-.0305) 56.56,55
56  ARAO=XRAO
55  CONTINUE
   IF (ARAO) 30,57,57
57  CONTINUE
   SACH4=(1.-XP*(ARAO**5))/(2.*(XAL*CA-XLA))
   IF (SACH4) 53,53,54
54  CONTINUE
   PIPO=XPIPO*(1.-CA*SACH4*SACH4)**CB
   PTSPU=PTIPO
   SACH5=(CE*(1.-((PTSPU**CQ))**5
   ARAO=(1.-CA*(SACH5**2))**CJ /SACH5
   WHMWJ UX, WHCPM UX, WHCOM UX, WHGMA /)
76  CONTINUE
55  SACH2=SACH2

```

[illegible]



FIXED AREA HEATED JET PUMP, VARIABLE DISCHARGE PRESSURE

P1P0	T1T0	A2A3	SACH2	SACH3	SACH4	SACH5	A3A6	A5A6	TT5T0	PT4P0	P4P0	MMNJ	CPM	COM	CMA
1.600	1.000	2.160													
P15P0	SACH2	SACH3													
1.272	.000	.668			.254	.632	.812	1.192	1.000	1.272	1.225	.000	1.365	-1.365	.804
1.270	.010	.668			.260	.630	.794	1.194	1.000	1.270	1.221	.022	1.350	-1.350	.800
1.268	.020	.669			.266	.628	.776	1.197	1.000	1.268	1.217	.043	1.317	-1.317	.796
1.266	.030	.669			.273	.626	.759	1.199	1.000	1.266	1.212	.065	1.294	-1.294	.792
1.264	.040	.670			.279	.624	.742	1.202	1.000	1.264	1.208	.087	1.272	-1.272	.789
1.262	.050	.670			.286	.622	.727	1.204	1.000	1.262	1.203	.109	1.251	-1.251	.785
1.260	.060	.671			.292	.620	.711	1.207	1.000	1.260	1.199	.130	1.231	-1.231	.781
1.259	.070	.671			.299	.618	.697	1.209	1.000	1.259	1.194	.152	1.212	-1.212	.778
1.257	.080	.672			.305	.616	.683	1.212	1.000	1.257	1.190	.173	1.193	-1.193	.774
1.255	.090	.672			.312	.614	.670	1.214	1.000	1.255	1.185	.195	1.175	-1.175	.771
1.253	.100	.673			.319	.612	.657	1.217	1.000	1.253	1.180	.216	1.158	-1.158	.767
1.251	.110	.674			.326	.610	.644	1.219	1.000	1.251	1.176	.238	1.142	-1.142	.764
1.250	.120	.676			.332	.608	.632	1.222	1.000	1.250	1.171	.259	1.126	-1.126	.760
1.248	.130	.677			.339	.607	.621	1.224	1.000	1.248	1.166	.280	1.110	-1.110	.757
1.246	.140	.678			.346	.605	.610	1.226	1.000	1.246	1.161	.301	1.096	-1.096	.754
1.245	.150	.680			.353	.603	.599	1.229	1.000	1.245	1.157	.322	1.081	-1.081	.751
1.243	.160	.681			.360	.602	.589	1.231	1.000	1.243	1.152	.343	1.068	-1.068	.748
1.242	.170	.683			.367	.600	.579	1.234	1.000	1.242	1.147	.364	1.054	-1.054	.745
1.240	.180	.684			.374	.596	.569	1.236	1.000	1.240	1.142	.384	1.041	-1.041	.742
1.239	.190	.686			.381	.597	.560	1.238	1.000	1.239	1.137	.405	1.029	-1.029	.739
1.237	.200	.688			.388	.595	.551	1.240	1.000	1.237	1.132	.425	1.017	-1.017	.736
1.236	.210	.690			.395	.593	.542	1.243	1.000	1.236	1.127	.445	1.005	-1.005	.733
1.234	.220	.692			.402	.592	.534	1.245	1.000	1.234	1.122	.466	.994	-.994	.730
1.233	.230	.695			.410	.590	.526	1.247	1.000	1.233	1.116	.486	.983	-.983	.728
1.232	.240	.697			.417	.589	.518	1.249	1.000	1.232	1.111	.505	.973	-.973	.725
1.230	.250	.699			.424	.587	.510	1.252	1.000	1.230	1.106	.525	.963	-.963	.723
1.229	.260	.702			.431	.586	.503	1.254	1.000	1.229	1.101	.545	.953	-.953	.720
1.228	.270	.704			.439	.584	.496	1.256	1.000	1.228	1.095	.564	.943	-.943	.718
1.226	.280	.707			.446	.583	.489	1.258	1.000	1.226	1.090	.583	.934	-.934	.715
1.225	.290	.710			.453	.582	.483	1.260	1.000	1.225	1.084	.602	.925	-.925	.713
1.224	.300	.712			.461	.580	.476	1.262	1.000	1.224	1.079	.621	.917	-.917	.710
1.223	.310	.715			.469	.579	.470	1.264	1.000	1.223	1.073	.640	.908	-.908	.708
1.222	.320	.718			.476	.578	.464	1.266	1.000	1.222	1.068	.658	.900	-.900	.706
1.221	.330	.722			.483	.576	.459	1.268	1.000	1.221	1.062	.676	.892	-.892	.704
1.220	.340	.725			.491	.575	.453	1.270	1.000	1.220	1.057	.694	.885	-.885	.701
1.218	.350	.728			.498	.574	.448	1.272	1.000	1.218	1.051	.712	.878	-.878	.699
1.217	.360	.731			.506	.573	.442	1.274	1.000	1.217	1.045	.730	.870	-.870	.697
1.216	.370	.735			.513	.571	.437	1.276	1.000	1.216	1.039	.748	.863	-.863	.695
1.215	.380	.738			.521	.570	.432	1.278	1.000	1.215	1.034	.765	.857	-.857	.693
1.214	.390	.742			.528	.569	.428	1.280	1.000	1.214	1.028	.782	.850	-.850	.691
1.213	.400	.745			.536	.568	.423	1.281	1.000	1.213	1.022	.799	.844	-.844	.689
1.213	.410	.749			.544	.567	.419	1.283	1.000	1.213	1.016	.816	.838	-.838	.687
1.212	.420	.753			.551	.566	.414	1.285	1.000	1.212	1.010	.832	.832	-.832	.686
1.211	.430	.757			.559	.565	.410	1.287	1.000	1.211	1.004	.849	.826	-.826	.684
1.210	.440	.761			.567	.564	.406	1.288	1.000	1.210	.998	.865	.820	-.820	.682
1.209	.450	.765			.574	.563	.402	1.290	1.000	1.209	.992	.881	.815	-.815	.680
1.208	.460	.769			.582	.562	.399	1.292	1.000	1.208	.986	.897	.810	-.810	.679
1.207	.470	.773			.590	.561	.395	1.293	1.000	1.207	.980	.912	.804	-.804	.677

1.206	.480	.977	.597	.560	.391	1.295	1.003	1.206	.973	.927	.799	-.799	.675
1.206	.490	.982	.605	.559	.388	1.297	1.000	1.206	.967	.943	.794	-.794	.674
1.205	.500	.986	.613	.558	.385	1.298	1.000	1.205	.961	.958	.790	-.790	.672
1.204	.510	.991	.620	.557	.382	1.300	1.000	1.204	.955	.972	.785	-.785	.671
1.203	.520	.995	.628	.556	.379	1.301	1.000	1.203	.948	.967	.781	-.781	.669
1.203	.530	1.000	.636	.555	.376	1.303	1.000	1.203	.948	1.001	.776	-.776	.668
1.202	.540	1.000	.644	.554	.373	1.304	1.000	1.202	.936	1.015	.772	-.772	.666
1.201	.550	1.000	.651	.553	.370	1.306	1.000	1.201	.929	1.029	.768	-.768	.665
1.201	.560	1.000	.659	.553	.367	1.307	1.000	1.201	.923	1.043	.764	-.764	.663
1.200	.570	1.000	.667	.552	.365	1.309	1.000	1.200	.917	1.056	.760	-.760	.662
1.199	.580	1.000	.674	.551	.362	1.310	1.000	1.199	.910	1.069	.756	-.756	.661
1.199	.590	1.000	.682	.550	.360	1.311	1.000	1.199	.904	1.082	.753	-.753	.660
1.198	.600	1.000	.690	.550	.358	1.312	1.000	1.198	.897	1.095	.749	-.749	.659
1.198	.610	1.000	.698	.549	.355	1.314	1.000	1.198	.891	1.107	.746	-.746	.657
1.197	.620	1.000	.705	.548	.353	1.315	1.000	1.197	.884	1.119	.742	-.742	.656
1.197	.630	1.000	.713	.548	.351	1.317	1.000	1.197	.878	1.131	.739	-.739	.655
1.196	.640	1.000	.721	.547	.349	1.318	1.000	1.196	.871	1.142	.736	-.736	.654
1.196	.650	1.000	.729	.546	.347	1.318	1.000	1.196	.864	1.153	.733	-.733	.653
1.195	.660	1.000	.737	.546	.345	1.319	1.000	1.195	.858	1.164	.730	-.730	.653
1.195	.670	1.000	.744	.545	.343	1.320	1.000	1.195	.851	1.175	.727	-.727	.652
1.194	.680	1.000	.752	.545	.342	1.321	1.000	1.194	.845	1.185	.724	-.724	.651
1.194	.690	1.000	.760	.544	.340	1.322	1.000	1.194	.838	1.195	.722	-.722	.650
1.194	.700	1.000	.768	.544	.338	1.322	1.000	1.194	.831	1.205	.719	-.719	.649
1.193	.710	1.000	.776	.544	.337	1.323	1.000	1.193	.825	1.214	.716	-.716	.649
1.193	.720	1.000	.783	.543	.335	1.324	1.000	1.193	.818	1.223	.714	-.714	.648
1.193	.730	1.000	.791	.543	.334	1.325	1.000	1.193	.811	1.232	.712	-.712	.647
1.192	.740	1.000	.799	.542	.333	1.325	1.000	1.192	.804	1.241	.710	-.710	.647
1.192	.750	1.000	.807	.542	.331	1.326	1.000	1.192	.798	1.249	.707	-.707	.646
1.192	.760	1.000	.815	.542	.330	1.326	1.000	1.192	.791	1.257	.705	-.705	.646
1.192	.770	1.000	.822	.542	.329	1.327	1.000	1.192	.784	1.264	.703	-.703	.645
1.191	.780	1.000	.830	.541	.328	1.327	1.000	1.191	.777	1.271	.701	-.701	.645
1.191	.790	1.000	.838	.541	.327	1.328	1.000	1.191	.771	1.278	.700	-.700	.644
1.191	.800	1.000	.846	.541	.326	1.328	1.000	1.191	.764	1.285	.698	-.698	.644
1.191	.810	1.000	.854	.541	.325	1.329	1.000	1.191	.757	1.291	.696	-.696	.644
1.191	.820	1.000	.861	.540	.324	1.329	1.000	1.191	.750	1.297	.695	-.695	.643
1.191	.830	1.000	.869	.540	.323	1.329	1.000	1.191	.743	1.303	.693	-.693	.643
1.191	.840	1.000	.877	.540	.322	1.330	1.000	1.191	.737	1.308	.692	-.692	.643
1.190	.850	1.000	.885	.540	.322	1.330	1.000	1.190	.730	1.313	.690	-.690	.643
1.190	.860	1.000	.892	.540	.321	1.330	1.000	1.190	.723	1.318	.689	-.689	.642
1.190	.870	1.000	.900	.540	.320	1.330	1.000	1.190	.716	1.323	.688	-.688	.642
1.190	.880	1.000	.908	.540	.320	1.330	1.000	1.190	.709	1.327	.687	-.687	.642
1.190	.890	1.000	.916	.539	.319	1.331	1.000	1.190	.703	1.330	.686	-.686	.642
1.190	.900	1.000	.923	.539	.319	1.331	1.000	1.190	.696	1.334	.685	-.685	.642
1.190	.910	1.000	.931	.539	.318	1.331	1.000	1.190	.689	1.337	.684	-.684	.642
1.190	.920	1.000	.939	.539	.318	1.331	1.000	1.190	.682	1.340	.683	-.683	.642
1.190	.930	1.000	.947	.539	.317	1.331	1.000	1.190	.676	1.342	.683	-.683	.642
1.190	.940	1.000	.954	.539	.317	1.331	1.000	1.190	.669	1.344	.682	-.682	.642
1.190	.950	1.000	.962	.539	.317	1.331	1.000	1.190	.662	1.346	.682	-.682	.642
1.190	.960	1.000	.970	.539	.317	1.331	1.000	1.190	.655	1.347	.681	-.681	.642
1.190	.970	1.000	.977	.539	.317	1.331	1.000	1.190	.649	1.349	.681	-.681	.642
1.190	.980	1.000	.985	.539	.317	1.331	1.000	1.190	.642	1.349	.681	-.681	.642
1.190	.990	1.000	.993	.539	.316	1.331	1.000	1.190	.629	1.350	.681	-.681	.642
1.190	1.000	1.000	1.000	.539	.316	1.331	1.000	1.190	.629	1.350	.681	-.681	.642

# INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2. Library U. S. Naval Postgraduate School Monterey, California	2
3. Commander, Naval Air Systems Command Department of the Navy Washington, D. C. 20360	1
4. Dr. Richard W. Bell (Chairman) Department of Aeronautics U. S. Naval Postgraduate School, Monterey, California	2
5. Professor Theodore H. Gawain Department of Aeronautics U. S. Naval Postgraduate School, Monterey, California	4
6. LT Carl R. Summers, USN 1315 South Third Street Aberdeen, South Dakota 57401	1
7. LT Wendell C. Ridder R. R. #1 Higginsville, Missouri 64037	1





## DOCUMENT CONTROL DATA - R&amp;D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U. S. Naval Postgraduate School Monterey, California 93940		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE Theoretical Analysis of the Heated Jet Pump and Design of a Test Facility			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) N/A			
5. AUTHOR(S) (Last name, first name, initial) RIDDER, Wendell C., Lieutenant, U. S. Navy SUMMERS, Carl R., Lieutenant, U. S. Navy			
6. REPORT DATE May 1966	7a. TOTAL NO. OF PAGES 100 113	7b. NO. OF REFS eight	
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)  N/A		
b. PROJECT NO.  N/A			
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
d.	N/A		
10. AVAILABILITY/LIMITATION NOTICES <del>Unclassified requesters may obtain copies of this report from DDC.</del>			
11. SUPPLEMENTARY NOTES  None		12. SPONSORING MILITARY ACTIVITY U. S. Naval Postgraduate School Monterey, California 93940	
13. ABSTRACT <p>The jet pump is a device that compresses low-pressure fluid by means of a high energy jet. This device is simple, reliable and light weight, and is attractive for various uses including eventual application to boundary layer control for aircraft.</p> <p>In this study the theoretical flow of a perfect gas through constant pressure and constant area jet pumps is predicted by analyzing the equations of continuity, energy and momentum. Of particular interest is the effect of heating the high energy jet. The parameters describing optimum heated jet pumps are determined. The ideal pumps presented herein represent upper performance limits for actual devices.</p> <p>Also included is a complete description and design of a facility for testing heated jet pumps. Specific test configurations are analyzed, and performance curves are illustrated. These curves facilitate comparison between theory and experiment.</p>			

14.	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
	Jet						
	Pump						
	Heated						
	Boundary Layer						
	Flow						
	Mixing						
	Compressible						
	Subsonic						
	Tube						
	Design						
	Control						
	Compressor						

### INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.
- 2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.
- 7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.
- 8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.
- 9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).
10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through \_\_\_\_\_."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through \_\_\_\_\_."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through \_\_\_\_\_."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical content. The assignment of links, roles, and weights is optional.











11

mesR45

DUDLEY KNOX LIBRARY



3 2768 00416380 8

DUDLEY KNOX LIBRARY